# VISUAL SLANT UNDERESTIMATION

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by

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Note:

Part of the material in this thesis has been published in Perception, 1980 (9), 285-302, under the title of "Slant Underestimation: A model based on the size of the viewing aperture". This article was prepared at an early stage of my thesis research and some of the ideas incorporated in the paper have been modified or abandoned altogether. If a conflict of ideas exists between the material in the above paper and the material contained in this thesis, the latter is to be taken as my most recent viewpoint.

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# ABSTRACT

A general model of visual slant underestimation is presented. It is based on the idea that two specific types of perceptual error occur in the evaluation of the slant angle by the observer. The reason for these errors occurring is postulated to be that reduced viewing conditions result in the deviation of the observer's perceived straight-ahead direction from the true direction. Specifically this deviation is postulated to be in the direction of the nearest part of the surface in accord with conditions that exist in our everyday environment. In the case of a slanted rectangle, correct registration of the projected length of half of the surface and the correct registration of the angle of convergence, will result in perception being veridical. A mechanism is outlined which indicates how both of these factors are misperceived and an equation is developed which enables the predicted slant estimates to be calculated, given the dimensions of the rectangle and its distance from the eye. Equations for the case of slanted surfaces viewed through apertures are also developed. The model is assessed in relation to past slant perception experiments and is found to be in close agreement with the empirical results. Four new experiments are reported which test specific predictions of the model and it is concluded that the model is a good predictor of the large amount of previously unexplained underestimation that occurs in slant perception studies.

# CHAPTER I

## GENERAL INTRODUCTION

Slant is a visual impression which can be defined as the angular inclination of a surface to the line of regard. By the geometry of perspective, a physical surface not perpendicular to the line of sight does of necessity project a distribution of texture elements which get denser as the surface recedes and whose gradient of density is proportional to the angle of slant. With this fact in mind, Gibson (1950a) proposed that "the apparent slant of a surface to the line of regard at any point might be given by the rate of increase of the density of elements at the corresponding point in the image". This was an example of Gibson's (1950a, b) general hypothesis of psychophysical correspondence, as an approach to the problem of visual space perception. His theory hypothesised a one-to-one correspondence between perception and the geometric parameters of the optical stimulus. In this particular case (Gibson, 1950a), the geometric parameter he examined, was the property of gradient of texture density. This is the gradual increase in the density of the fine structure, the spots or gaps, or the extended pattern of either a part or the whole of the visual field.

To test his hypothesis, Gibson (1950a) devised an experimental situation which allowed the texture density to vary while eliminating factors such as disparity, binocular convergence, movement parallax and linear perspective. The first three factors were eliminated by restricting subjects to monocular viewing and by preventing head movements. Differential focus of the near and far parts of the surface was controlled by using projected photographic slides instead of real surfaces and linear perspective or outline convergence was eliminated by having the subjects view the surfaces through an aperture that masked the edges of the surface.

Subjects were required to judge the slant of the apparent surfaces by setting a response device at the same inclination from the frontal plane. Gibson (1950a) found that the gradient of texture density was in psychophysical correspondence with the property of optical slant, but he also found, as did many researchers after him, that there was a tendency for the subjects to underestimate the absolute degree of slant.

More recently, Gibson (1979) modified his earlier position regarding this approach. He admitted that phenomenal slant does not simply correspond to the gradient. This he claimed was shown by the large group of experiments which report that apparent slant is not equal to the geometrically predicted slant, but rather it is always less than it should be theoretically. Gibson (1979) concluded that the impression of slant cannot be isolated and experimentally manipulated by displaying a texture inside a window because "the perception of the occluding edge of the window will affect it". He claimed that slant is not an absolute quantity as he originally conceived it to be but rather it is always relative. Therefore we cannot, according to Gibson (1979), isolate the impression of slant by displaying a texture inside a window because the surfaces will be slanted relative to the surface that has the window in it. The experimental results show that the angular separation of these two surfaces is underestimated.

This underestimation has never really been fully explained and the underestimation of slant has rarely been examined in its own right. Rather, it has always been an awkward consequence of the attempt to obtain veridical slant judgements through the introduction of various surface parameters such as the size of the texture elements.

Gibson (1979) discounted his original experimental paradigm because it studied optical slant, not geographical slant, and it was not appropriate for testing his theory of the direct perception of surface layout (Gibson 1950a,b). The fact remains that optical slant is underestimated under the conditions used by Gibson (1950a) and a large number of researchers after him. Viewing a slanted surface through an aperture with an upright fixed head position, and monocular viewing, results in the slant of the surface being underestimated. This thesis begins by considering this underestimation to be of primary interest. Rather than considering the traditional problem of finding the determinants of slant perception, it attempts to find out why slant perception breaks down in this particular experimental set-up. Many modern day optical systems possess some of the properties that were characteristic of traditional slant perception experiments; namely monocular viewing, fixed upright head position and limited field of view, e.g. videoscreens, telescopes, periscopes, flight simulators, and aircraft head-up displays. It is of interest therefore to examine this particular perceptual error and attempt to determine the conditions under which it occurs.

## 1.1 Experimental Evidence of Underestimation

Some insight can be gained as to the possible factors producing underestimation, by examining some of the past experiments on slant perception. A wide variety of experimental conditions have been used in these studies and many variables have been tested.

Gibson (1950a) was the first to introduce texture as a factor in slant perception experiments. He used two conditions; a regular texture condition which consisted of a brick-like pattern and an irregular texture condition which was made up of an uneven distribution of elements. Both types were obtained by photographing wallpaper patterns at various angles. Gibson tested his subjects over eight slant angles and the subject viewed the test surfaces through a circular aperture arrangement which gave a 24° field of view. Their task was to duplicate the perceived slant of the test surface on a response device. The conditions and results can be summarized as follows:

| Actual Slant (backward) | Regular Texture Judged Slant | Irregular Texture Judged Slant |
|---|---|---|
| 10° | -0.8° (forward) | 6.4 |
| 22° | 8.6° | 7.8° |
| 30° | 18.9° | 9.9° |
| 45° | 25.3 | 23.9° |
| Actual Slant (forward) | | |
| 10° | 8.5° | 8.6° |
| 22° | 15.9° | 7.7° |
| 30° | 21.9° | 9.2° |
| 45° | 28.6° | 17.9° |

Backward slant is the case where the top part of the surface is slanted away from the observer and forward slant is the case where the top is slanted towards the observer.

Gibson's (1950a) subjects showed less underestimation for the forward slant condition than for the backward slant condition in the regular texture series. Gibson explained this in terms of the baseline used for his response device, a constant error being introduced presumably in the positive direction. However, this does not account for the fact that this discrepancy does not show up in Gibson's (1950a) irregular texture condition, which used the same response device.

The irregular texture surfaces generally produced poorer slant judgements than the regular texture condition, although for some angles the judgements seem to have improved. Gibson (1950a) concluded that a regular texture affords a more definite gradient of density than does an irregular texture. In the same paper Gibson reported that he had shown a psychophysical correspondence between the gradient of texture density and the property of optical slant, although the correspondence was not perfect.

Gibson (1950a) noted that the discrepancy between the judged slant and the physical slant equivalent to the gradient was consistently in the direction of a frontal surface. He thus explained the underestimation by saying that the judged slants were compromises. He claimed that factors such as the visibility of the texture of the screen onto which the test surface slides were projected, together with the absence of differential blur and the resulting absence of the cue of accommodation would favour a compromise between a slanted and a frontal impression. Other factors he also felt contributed to this 'frontal tendency' was the tendency of perception to conform to the slant of the hole-screen (aperture of reduction screen) and any residual head-motion not eliminated by the headrest used in the experiment. The former refers to an effect first pointed out by Katz (1935) in which uniform fields of colour viewed through an aperture appear to fill in the hole and take on the same slant (frontal) as the hole-screen.

The majority of reasons put forward by Gibson (1950a) for the 'frontal tendency' or underestimation are based on the fact that a two-dimensional representation of the surface was used. Gruber and Clark (1956) in a study aimed at examining specific properties of texture gradients that affect slant, used a real surface which was pivoted to various angles of slant. The surface was pivoted around a vertical axis, so the slant was 'side to side' rather than backwards and forwards as in Gibson's (1950a) case. Even though the subjects viewed a real surface through an aperture, Gruber and Clark's (1956) subjects still showed a large constant error, always in the direction of the frontal-parallel plane. Arguments put forward by Gibson (1950a) involving texture of the projection screen, or absence of the cue of accommodation do not stand up in the Gruber and Clark (1956) case. The best performance by Gruber and Clark's (1956) subjects resulted from the use of 'large dots' or a coarse texture, viewed from a distance of 1.5m. However, the mean judgement for actual slants of 32°, 43° and 53° were 12.5°, 18.0° and 24.2° respectively. Relative to small coarse texture units, judgements of slant were significantly greater for large coarse texture units. Although they were able to show that changing the composition of the texture on the test surface influenced the impression of slant, Gruber and Clark (1956) offered no explanation as to why, for even their most favourable condition, slant was underestimated by up to 28°.

Clark, Smith and Rabe (1956a) carried out a slant perception experiment in which several different stimulus conditions were used. One of these conditions consisted of a textured surface consisting of regularly spaced white circular dots on a dark field. This surface was pivoted about a vertical axis and viewed through a circular reduction screen aperture. When the surface was set at 40°, the mean judged slant based on 96 observations was a mere 9.7°. Their experiment was designed to compare the effectiveness of outline distortion against texture gradients as sources of information for slant perception. As in the case of Gruber and Clark (1956), Clark, Smith and Rabe (1956a) were interested in the relative performances of their subjects under the different experimental conditions, and offered no explanation for the large underestimation that occurred in the texture condition.

Flock and Moscatelli (1964) tested six different surface textures with differing degrees of irregularity. Subjects were tested over nine different slant angles. Unfortunately Flock and Moscatelli (1964) report their results in terms of regression coefficients. A value of 1.0 implies maximal accuracy for slant estimates. The highest mean regression coefficient was .78 for regularly shaped texture elements, which were irregularly distributed over the surface.

Braunstein (1968) used computer generated pictures of random dot patterns on slanted planes to test the effect of motion and texture as sources of slant information. His condition with zero velocity fits into the paradigm of Gibson's (1950a) original slant experiment. For test surfaces representing 0°, 20°, 40° and 60°, Braunstein (1968) obtained mean slant judgements of -0.3°, 2.1°, 1.8° and 11.6° respectively, from which he concluded that texture gradients appeared quite insufficient as sources of slant information in his study. Certainly, Braunstein's (1968) subjects exhibited the largest reported degree of underestimation for this type of experiment, and one is led to suspect some aspect of his design as causing such large perceptual errors.

Underestimation is apparent in all the studies involving texture density. These studies include those using projected two-dimensional displays as well as real surfaces; surfaces rotated around a horizontal axis and surfaces rotated around a vertical axis. A wide variety of response devices have been used ranging from tactual kinaesthetic palm-boards (e.g. Gibson, 1950a), to pivoting boards which are set visually (e.g. Gruber and Clark, 1956). Underestimation occurs over all angles although some results suggest that it is greater for some angles than for others.

Not long after Gibson's (1950a) experiment, researchers began to test whether outline perspective was an adequate stimulus for slant. Linear perspective involves the compression or convergence of the outlines of a figure. Gibson (1950a), by using an aperture, had specifically avoided including the edges of his surfaces in his stimuli. He was trying to isolate the property of texture density. The experiments involving the outline of rectangular or circular shapes are closely related to studies testing the shape-slant invariance hypothesis, several of which preceded Gibson's (1950a) experiment. When a figure or the plane surface of an object is orientated obliquely to the line of sight, the shape of its retinal image differs from its real shape; if the perceived shape nevertheless approximates the objective shape, it is referred to as an example of 'shape-consistency'. Koffka (1935) offered an explanation of shape-constancy based on the assumption of an invariant linkage between slant and shape. The invariance hypothesis requires that perceived shape vary as a strict function of variations in perceived slant. This implies that if the slant of an object is underestimated, its perceived shape will regress towards its retinal shape. Generally experiments testing the shape-slant invariance hypothesis have failed to find any conclusive evidence for this (e.g. Beck and Gibson, 1955; Epstein, Botrager and Park, 1962; Clark, Smith and Rabe, 1955, 1956a, 1956b; Smith, 1964). These experiments are often similar to slant perception experiments and also exhibit underestimation, even though the test shape does not fully extend over the area of the aperture, as is the case with the pure slant experiments. The factors causing the underestimation in shape-slant experiments may or may not be the same as those in the aperture slant experiments, but they still warrant examination.

Clark, Smith and Rabe (1955) carried out an experiment to determine whether or not monocular gradient of outline convergence is, like texture gradient, a sufficient stimulus for the perception of slant. The stimulus forms consisted of white rectangles against a black background. They were slanted by rotation about a vertical axis. Clark, Smith and Rabe (1955) found that the mean perceived slant increased generally with increasing physical slant and they concluded that retinal gradient of outline convergence was a sufficient stimulus for slant, in accordance with the theory of psychophysical correspondence. They noted that the mean perceived slants were substantially less than the corresponding physical slants and pointed out that this was a common finding in their experiments. Clark, Smith and Rabe (1955) speculated that the discrepancy was probably due to the absence of other cues for depth; an unusual conclusion considering the fact that they were attempting to show that monocular gradients were a sufficient stimulus for the perception of slant, and that they had concluded that their results supported this hypothesis.

For physical slants of 0°, 20° and 40°, Clark, Smith and Rabe (1955) obtained mean judgements of 0.14°, 6.96° and 16.96° respectively, for a long rectangle (28cm x 15.4cm) and -0.57°, 4.89° and 17.32° for a short rectangle (19cm x 15.4cm). Follow-up experiments by Clark, Smith and Rabe (1956a, b) and Smith (1956, 1959, 1964, 1966) also exhibited large amounts of underestimation. So underestimation of slant is also apparent in the type of experiment in which a rectangle is slanted against a featureless background.

## 1.2 Factors Affecting Slant Judgements

Specific attempts at explaining underestimation per se, are rare in the slant perception literature. Explanations by Gibson (1950a) and Clark, Smith and Rabe (1955b) have already been mentioned. These explanations do not hold up in the light of the empirical evidence, and are not specific enough to enable us to make a quantitative estimate of the amount of underestimation expected in a given situation. This latter ability should be the goal of any adequate theory of the causes of underestimation. What the slant perception literature does offer us, is an insight into the relative efficacy of various factors upon slant judgements.

Clark, Smith and Rabe (1956) carried out an experiment in which the separate influence on accuracy of slant estimates of figure texture and outline convergence was demonstrated. Their results showed that outline convergence was apparently the more potent cue. The combination of texture and outline resulted in no better performance than did outline alone. In a subsequent experiment, Clark, Smith and Rabe (1956b) investigated the separate and joint influence of texture, outline and retinal disparity on slant estimation. In this study the figures were circular in shape, yielding elliptical retinal images when slanted. Disparity was manipulated by using monocular and binocular viewing. Once again the absolute value of slant was underestimated, and as before, outline was a more powerful cue than texture and the combination of outline and texture did not improve accuracy over outline alone. Binocular viewing led to more accurate estimates of slant than did monocular viewing, but underestimation was still evident.

Comparison of results between Clark et al's (1956b) study and their previous study (1956a) indicates that the slant of circles is more accurately perceived than the slant of rectangles. This was confirmed in a study by Smith (1956) which demonstrated the superiority of perceiving circles over perceiving rectangles for angles of slant greater than 30°. No conclusive reasons were presented, as to why this should be so. The greater variability in the possible forms of rectangles compared to circles, is just one of the differences between rectangles and circles that could be considered.

Gruber and Clark (1956) found that certain combinations of element size and inter-element separation were optimal for accuracy of slant estimation. Thus, with element size held constant, a separation among elements of 30mm yielded more accurate judgements than separations of either 6mm or 90mm. Eriksson (1964) obtained similar results and such data were used as an argument against the theory that texture density by itself could account for the perception of slant (Dember 1961; Eriksson 1964). Gruber and Clark (1956) argued that the maximal utilization of texture as a distance cue occurs at intermediate levels of density. When elements are too widely dispersed, they do not form a perceptual surface. When they are too tightly packed and too small, articulation of the elements is poor, and relative density loses its effectiveness. To be maximally useful as a cue to slant, the texture of the surface must be discriminable, but at the same time the density must not be so coarse as to interfere with the subject's ability to perceive the texture as an integral part of the surface, rather than a superficial aspect not conveying any slant information. However, if gradients of texture produced accurate perception of slant, then changing the size of the elements and their spacing should not theoretically affect the perceived slant, since the gradient remains the same. This is what Eriksson (1964) argued.

Flock (1965) raised objections to Gruber and Clark's (1956) experiment. He pointed out that in their condition for close separation of elements, the use of 6mm dots and a 6mm separation between dots, meant that the dots would have been in contact with each other, leaving an interspace where the dots met of approximately 1.5mm in height and shaped like an equilateral triangle. Flock (1965) argued that at the distances used by Gruber and Clark (1956), such a surface would appear homogeneous and untextured, resulting in the poor performance of the subjects for this condition. Flock (1965) also pointed out that at the other extreme, when dot separation was 90mm, only 20 dots over the entire display would have been visible. Flock (1965) said that most of the criticisms leveled against Gruber and Clark (1956) apply equally to Eriksson (1964).

Gruber and Clark (1956) also varied the distance from the subject's eye to the test surface. The effect of distance upon the estimates of slant was found to be significant, with the greatest accuracy occuring at close distances. This was evident for test surfaces not susceptible to Flock's (1965) criticisms relating to the surface appearing homogeneous and uniform to the subjects, although an improvement in judgements for closer distances would be expected in such a case. Gruber and Clark (1956) concluded that the main effect of varying the distance of observation was to vary simultaneously the retinal size and density of the units composing the texture surface and therefore they ignored the distance variable as such, and concentrated on the variables of unit size and density.

Smith (1956) in an experiment designed to determine differences in perceptual error for circles and rectangles at several angles of slant, noticed that the percentage error in perception (i.e. the difference between the actual and perceived slants expressed as a percentage of the actual slant), decreased regularly as the angle of slant increased for angles greater than zero. The relationship between error and angle was not a simple function however, and depended on whether a circle or a rectangle was being used. One must also consider that a constant error arising from the response device or measuring technique, could produce a larger percentage error for the small actual angles of slant, than for the larger angles. Gibson's (1950a) data for the regular texture condition shows no obvious relationship between amount of error and the slant angle used. In fact, Gibson's data for his forward slant, regular texture condition, shows an increase in the percentage error as the slant angle increases, contrary to Smith's (1956) data. There does seem to be some indication that slant estimates are more accurate for particular angles, but insufficient studies exist, in which a wide range of slant angles have been used, to draw any definite conclusions.

Braunstein (1968), using computer generated pictures of random dot patterns on slanted planes, obtained mean slant estimates of -0.3°, 2.1°, 1.8° and 11.6° for actual slants of 0°, 20°, 40° and 60°. As was mentioned previously, these results represent some of the largest underestimates for this type of experiment. The fact that Braunstein (1968) used a two-dimensional projected display should not account for such large perceptual errors, since Gibson (1950a) also used this type of display. One interesting aspect of Braunstein's (1968) procedure is that he effectively modified the vertical extent of the surface visible to the subject, depending on the particular angle of slant. This is equivalent to using a large surface for large slant angles and a smaller surface for small slant angles. Braunstein realised that when using a restricted field of view, more of the vertical extent of the surface plane is visible when greater slant angles are used. This would mean that for his conditions in which a velocity gradient was present, a greater range of velocities would be visible for large slant angles, thus introducing an extraneous variable. Braunstein's (1968) computer generated display could thus be modified to control for the amount of vertical extent visible to the subject. No other researchers had altered the vertical extent of the test surfaces with different slant angles, and this could be a factor in producing the large perceptual errors.

An alternative explanation for Braunstein's (1968) results may be that his test surfaces consisted of a large number of very small dots, something he was able to achieve through the use of computer generated displays, but which were rarely used by researchers utilizing 'hand made' test surfaces. Flock (1964a) had used an electrostatically produced test surface, which was highly irregular and did not 'correspond to anything commonly experienced in the physical environment'. Flock (1964a) obtained a mean regression coefficient of only 0.13 for this surface. The indication seems to be that the "texture" surfaces used by several investigators (e.g. Gibson, 1950a; Gruber and Clark,1956) may not have completely isolated the variable of texture density. It is possible that some 'perspective information' is still visible in these patterns. It would seem that when the texture is truly random as in Braunstein's (1968) and Flock's (1964a) cases, the texture density gradient is only minimally effective as a source of slant information.

In the experiments in which a slanted rectangle is used to test slant judgements, there is some evidence that the size of the rectangle affects the accuracy of the judgement. This effect was first noticed by Stavrianos (1945), in a study on the shape-slant invariance problem. She used standard and comparison plane rectangles of different sizes to avoid the possibility that subjects could make a retinal match in equating slant. Even under unreduced conditions of observation, the results of the experiment indicate that subjects consistently saw the slant of the large rectangle as being greater than that of the smaller of the two rectangles. Freeman (1962, 1966a) replicated the Stavrianos effect under complete reduction conditions, and used such results to support his view that variables of surface texture were both ineffectual and unnecessary for the perception of slant. Freeman (1965) argued that all perceived slants are a function primarily of linear outline perspective and that the greater the linear perspective, the greater the judged visual slant. Freeman (1965) pointed out how perspective depends not only upon the slant angle of the surface but also on its height, width and distance from the eye. But Flock (1965) argued that in studies by Epstein (1962) and Freeman (1966c), in which they used smaller rectangles than other experimenters, markedly better slant judgements were obtained. It was as if the smaller the rectangle, the better the slant judgements became. This was contrary to Freeman's (1965) theory which suggested that slant judgements became greater as the rectangles are made larger.

The debate between Flock and Freeman (Flock, 1964c; Freeman, 1965; Flock, 1965; Freeman, 1965b) represents one of the richest areas in slant perception research in terms of an examination of the variables affecting slant judgements. Flock (1964c) elaborated on Gibson's (1950b) gradient concept and presented an analysis of the variables arising from textured slanted surfaces. He claimed to show that accurate monocular slant perceptions are possible, even though a motionless viewer has had no previous experience with a particular substance, even though the textural elements of a motionless surface are irregular in size, shape and separation and even though parts of the surface are illuminated differently. His analysis was primarily an ecological description of the optics of slant; a description of the transformations which light reflected from a real, planar surface undergoes in projection as it converges on the observer's eyes. Similar analyses had been carried out by Gibson, Olum and Rosenblatt (1955) and Purdy (1958).

Freeman (1965) criticised Flock's (1964c) paper and argued against the psychophysical viewpoint. Freeman's (1965) main argument against Flock's (1964c) texture theory was that it did not predict varying estimates of slant with changes to the field of view of the test surface. Freeman (1965) considered outline perspective to be the primary factor influencing the perception of slant. When textured surfaces were used, he proposed that certain properties of the visual texture acted as cues to slant under some stimulus conditions. One of the variables proposed by Freeman (1966b) was the projective difference in visual angle size of near and far texture elements. Another cue considered by Freeman (1966b) was monocular accommodation and image clarity (or lack of it). Also, when the same textured surface is presented repeatedly to the same subject at different slants, the total number of visible elements, according to Freeman (1966b) may well be a conditional cue to slant perception when surfaces are viewed through apertures of fixed visual angle. If other cues for direction of slant are effective, then the number of visible elements can provide a secondary cue to the amount of slant. Finally, Freeman (1966b) proposed that there may also be some perspective cues generated by the change in average angular separation of texture elements. This assumes that the visual system responds as if there were linear contours present whose retinal convergence could then be evaluated.

Freeman (1966b) argued that these variables are dependent upon the field of view of the test surface and so judged slant should increase with increased viewing area of a slanted surface. Freeman (1966b) mentions the data of Gibson (1950a) and Flock and Moscatelli (1964) as support for this prediction. However, a study by Eriksson (1964) not quoted by Freeman (1966b) showed that increasing the field of view, resulted in a decrease in the perceived slant. Certainly the field of view does seem to be one of the main variables affecting the slant judgements, although its exact role has not been clarified.

It is ironical that Freeman (1965, 1966a, b), working in opposition to Gibson's (1950a, b) psychophysical theory, provided some of the best analyses of the variables affecting slant judgements. Freeman (1966b) may have presented contrary evidence for Flock's (1964c) particular optical texture theory, but he ignores the fact that Gibson's (1950b) theory was not limited to texture density gradients, but that it also included perspective gradients. The variables described by Freeman (1966b) can easily be subsumed under a general category of gradients. However, Freeman (1966b) did not assume along with Gibson that vision is necessarily veridical, nor that the retinal image is isomorphic with the dimensions of object surfaces from which light is reflected into the eye. Freeman (1966b) noted that there are instances in which proximal stimulation results in non-veridical or inconsistent perceptual judgements. However, he went on to determine the effect of differential, verbal reinforcement on the effectiveness of certain cues in elliciting visual responses, and proposed a general theory of perceptual learning based on cue relevance. No specific predictions were made regarding the amount of underestimation expected under particular experimental conditions.

The above review is a summary of the variables and conditions which appear to affect slant judgements. A theory of underestimation would need to incorporate these factors and be able to account for specific results in given situations. The first step in the development of such a theory will be a geometrical analysis of the information reaching an observers eye from a surface slanted relative to the gaze line.

# CHAPTER II

## MATHEMATICAL ANALYSIS OF THE OPTICAL INFORMATION AT THE OBSERVERS EYE

Attempts at providing a mathematical description of the sheaf of light rays entering the eye, have been made by Gibson, Olum and Rosenblatt (1955), Purdy (1958), Flock (1964c), Freeman (1966a), Braunstein and Payne (1969), Phillips (1970), Lee (1974), Clocksin (1980), but not all of these have been specifically concerned with the case of monocular static slant perception. These studies typically consider a purely abstract mathematical situation in which the eye is reduced to a geometrical point, the earth to a plane, and the light reflected from the earth to lines intersecting in the point. The principle aim in many of these analyses has been to show that potential information exists in the light rays entering the eye, regarding the slant of the surface. There are many different measures that can be used for obtaining a relationship between the retinal information and the slant of the surface. Phillips (1970) lists seven such measures, just in relation to texture gradients. There are others for motion perspective (Gibson et al, 1955; Purdy, 1958; Lee, 1974; Clocksin, 1980) and for outline perspective (Freeman, 1966a; Braunstein and Payne, 1969).

No specific attempt has been made however to examine the information available to the subject in the slant perception experiment situation. A detailed geometrical analysis is required if we are to determine the factors causing slant misperception. This analysis needs to incorporate the factors which have been shown empirically to affect slant judgements, and not just to consist of an abstract mathematical perceptual model, as has been the case in many of the quoted studies.

The evidence presented above, indicates that outline perspective is more effective than texture gradients, so an analysis based on the perceived width of surface elements will be used. The analysis will be limited to the case of monocular slant perception to parallel the experiments quoted in Chapter I. Instead of considering the projection on to the retina we consider a surface projected onto a flat picture plane or projection plane, located in front of the eye position. There is a one-to-one correspondence between the points on such a plane and points on the retina and therefore information in the two projections is the same.

The first step is to derive a general equation for calculating the size of a given element on a surface, when it is projected onto the projection plane. From this we can examine the way in which these elements systematically change in size as the angle of slant of the surface changes.

Figure 1a shows the projection of a surface made up of regularly spaced elements $A_1A_2$, $B_1B_2$, $C_1C_2$ etc., each of width w, onto a projection plane, PP. Figure 1b is a cross-section through the points O, A and D. The angle of slant of the surface is labelled $\theta$. There are several different ways of defining this angle. First of all, if we consider a perpendicular to the surface at the point of fixation C (see figure 1b) then the angle of slant $\theta$ is the angle through which the line of regard OC, is rotated from the perpendicular, CN; ie. $\theta = \angle OCN$ in figure 1b.

An alternative definition is the angle through which the line of regard OC, has been moved from the perpendicular, OA, which extends from the

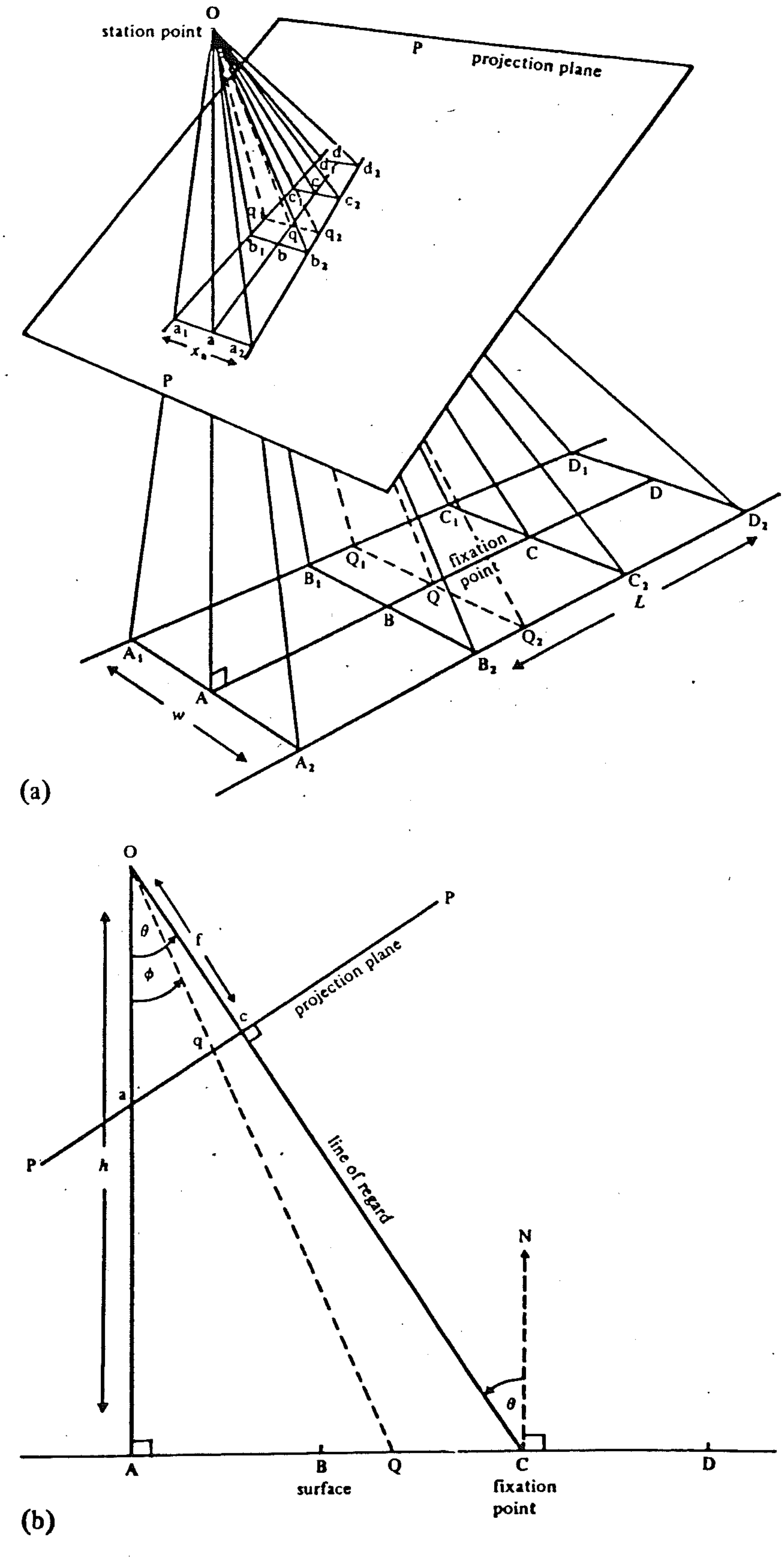



Figure 1. (a) Representation of the geometry involved in the projection of a surface onto a projection plane PP, when the station point is located at O; (b) cross-section through the points O, A, and D in figure 1a.

station point, O, to the surface; thus $\theta = \angle AOC$ in figure 1b. Note that $\angle OCN$ and $\angle AOC$ are geometrically equivalent since OA is parallel to CN.

A third way of defining the slant of the surface is to consider the angle at the intersection of the surface and an extension of the projection plane, PP, in the direction of the surface. A plane perpendicular to the line of regard, such as PP in figure 1b, lies at an angle $\theta$ to the surface, where $\theta$ is geometrically equivalent to $\angle AOC$ and $\angle OCN$. That is, if we let G be the point of intersection of the line PP (extended) and the surface in figure 1b, then $\angle DGP = \theta = \angle AOC = \angle OCN$. For convenience the following discussion will adopt the second definition as the slant angle $\theta$, namely, the angle between the line of regard and the perpendicular from O to the surface.

## 2.1 The Projected Size of an Element $Q_1Q_2$ on the Surface

We need to find the width $x_q$, of an element $Q_1Q_2$ on the surface when it is projected onto PP. In Figure 1b $\angle QOA = \phi$. Let $\nu = \theta - \phi$ The perpendicular distance from O to the projection plane is equal to f. This can be regarded as the 'focal length' of the system, and the size of f determines the 'magnification' of the projected image. The perpendicular distance from O to the surface is h. Let the actual width of element $Q_1Q_2$ be w.

The projected width of $Q_1Q_2$ relative to the actual width w, is proportional to the ratio of the length Oq to the total length OQ in figure 1b; i.e.

$$\frac{x_q}{w} = \frac{Oq}{OQ}$$

Now

$$Oq = \frac{f}{\cos\nu} \quad \text{and} \quad OQ = \frac{h}{\cos(\theta-\nu)}$$

therefore

$$\frac{Oq}{OQ} = \frac{f\cos(\theta-\nu)}{h\cos\nu}$$

When the right side of this equation is expanded, we obtain:

$$\frac{Oq}{OQ} = \frac{f(\cos\theta.\cos\nu + \sin\theta.\sin\nu)}{h}$$

therefore

$$x_q = \frac{f\cos\theta[1 + \tan\theta.\tan(\theta-\phi)]w}{h} \qquad [1]$$

This gives the projected width ($x_q$) of an element $Q_1Q_2$ on the surface for an angle AOQ = $\phi$ and an angle $\theta$ between the line of regard and the perpendicular from O to the surface. For example, for an element $A_1A_2$ on the surface at the base of the perpendicular OA, $\phi = 0°$, therefore,

$$x_q = \frac{f\cos\theta(1 + \tan^2\theta)w}{h} = \frac{fw}{h\cos\theta}$$

and for an element $C_1C_2$ at the point of fixation, $\phi = \theta$ and therefore

$$x_c = \frac{f\cos\theta w}{h}$$

Hence, given the values of $\theta$ and $\phi$, equation [1] simply enables us to calculate the projected width ($x_q$) of an element on the surface when the station point is a distance h from the surface and a distance f from the projection plane. It is now possible to examine how the relative sizes of different elements on the projection plane change according to the angle of slant $\theta$.

## 2.2 'Gradients' on the Projection Plane

Consider a rectangular unit $B_1B_2D_2D_1$ on the surface, of width w and length L (see figure 1a). The edge closest to A, the base of the perpendicular from O to the surface, is $B_1B_2$ and the opposite edge is $D_1D_2$.

This unit is projected onto PP as a regular trapezoid $b_1b_2d_2d_1$, with the bottom edge of width $x_b$ and the top edge equal to $x_d$, which is less than $x_b$. The height of the trapezoid is $l$ and it is equal to the length bd in figure 1a. This is the projected length of the unit.

This decrease in the width of the elements as one moves up the projection plane, is a form of gradient. This can be defined more precisely by letting the perspective gradient G = the difference between the projected width of an element on the surface $E_1E_2$ say, and the projected width of another element $F_1F_2$, divided by the projection of the distance between them. The elements $E_1E_2$ and $F_1F_2$ must be of the same width and must be centred on the line running from the base of the perpendicular to the point of fixation.

If one takes for example the two elements $Q_1Q_2$ and $C_1C_2$ on the surface, the centres of which are represented by Q and C in figure 1b, then from equation [1]

$$x_q = \frac{f\cos\theta(1+\tan\theta\cdot\tan\nu)w}{h}$$

and

$$x_c = \frac{f\cos\theta . w}{h}$$

therefore

$$x_q - x_c = \frac{f\sin\theta . \tan\nu . w}{h}$$

The distance between the projected elements, $q_1q_2$ and $c_1c_2$ is $l$ and it is equal to the length qc in figure 1b. Now $l = f\tan\nu$

therefore

$$G = \frac{x_q - x_c}{l} = \frac{f\sin\theta . \tan\nu . w}{h . f . \tan\nu}$$

i.e.

$$G = \frac{\sin\theta . w}{h} \qquad [2]$$

This is a general result and using the same procedure, it can be shown that for any two elements on the surface, the gradient of perspective between them is equal to $\sin\theta.w/h$ where $\theta$ the angle between the line of regard and the perpendicular from O to the surface, and h is the length of this perpendicular. In other words, the gradient of perspective is proportional to the angle of slant $\theta$.

An interesting aspect of this particular relationship is the presence of the two variables h and w. The gradient of perspective is also dependent upon how far from the surface the eye is, and the width of the original elements on the surface. A practical example of this can be seen by examining the perspective in two photographs, taken at different heights above a road or a set of railway lines.

## 2.3 Angle of Convergence

Equation [2] will now be extended to incorporate the important property of convergence or the angle of the perspective lines on the projection plane.

If figure 2 represents a portion of the information on a projection plane an arbitrary distance f from O, then G as defined above is equal to

$$\frac{x_1 - x_2}{l}$$

If we define an angle $\pi$ as the angle $q_1r_1$ makes with $q_1N$ ($q_1N$ is perpendicular to $q_1q_2$), then $\tan\pi = G/2$;

or

$$\tan\pi = \frac{\sin\theta.w}{2h} \qquad [3]$$

$\pi$ is the 'angle of convergence' of the lines on the projection plane and $\tan\pi$ is the 'gradient of perspective'. If D is the distance from the eye to

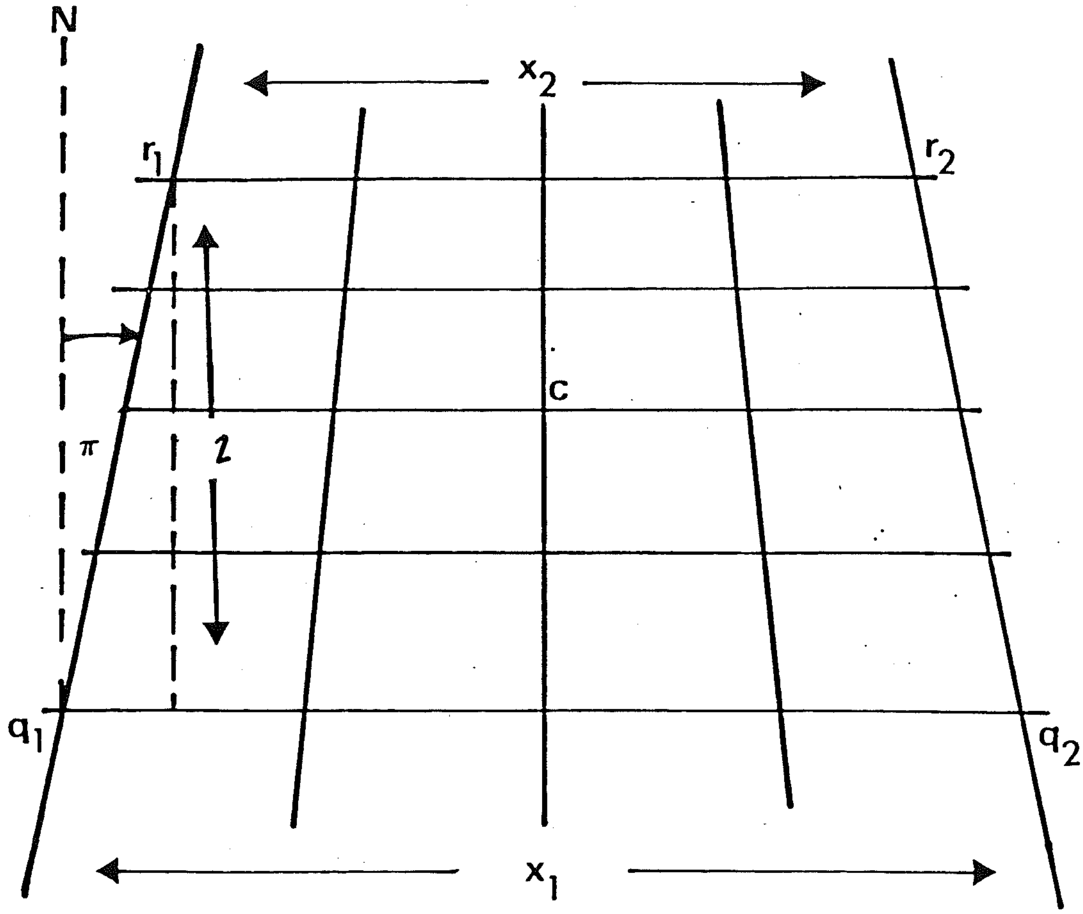



Figure 2. Representation of the information on a projection plane located an arbitrary distance f from the station point. The straight ahead direction passes through C on the surface and this is represented by c on the projection plane. $x_1$ and $x_2$ represent the lengths $q_1q_2$ and $r_1r_2$ respectively. The angle of convergence of the lines increases as we move out from c.

the fixation point on the surface, then $h = D\cos\theta$. Substituting this into [3] gives

$$\tan\pi = \frac{\tan\theta . w}{2D} \qquad [4]$$

If we move along the axis of rotation from the fixation point C a distance $^{w}/2$ then this length subtends an angle at the eye $\delta$, such that $\tan\delta = {}^{w}/_{2D}$ (As an example $\delta$, = $\angle COC_2$ in figure 1a).

Substituting this into [4] gives

$$\tan\pi = \tan\theta . \tan\delta$$

or

$$\tan\theta = \frac{\tan\pi}{\tan\delta} \qquad [5]$$

This is similar to an expression developed by Freeman's (1966a) but the derivation is different.

Given the two-dimensional information on the projection plane we can obtain the slant angle $\theta$ of the surface from the angle of convergence of perspective lines (or columns of dots) and the distance of the lines from the fixation point. For a given slant angle $\theta$, $\tan\pi$ increases or decreases, depending on the distance of the surface and the width of the particular elements. This has been pointed out by Freeman (1966a) and Braunstein (1968). In terms of the information on the projection plane, we can devise a simple 'optical device' for extracting the slant angle $\theta$ from the static two-dimensional information on a projection plane. See Figure 3.

We move up an arbitrary fixed distance Y on the projection plane, measured from the point of fixation C (or more correctly, the centre of the projection plane c). We then move out (either left or right) the same distance Y from c along a line perpendicular to cr. Finally we proceed across from r a number of units equal to that covered by cq, and call this distance X.

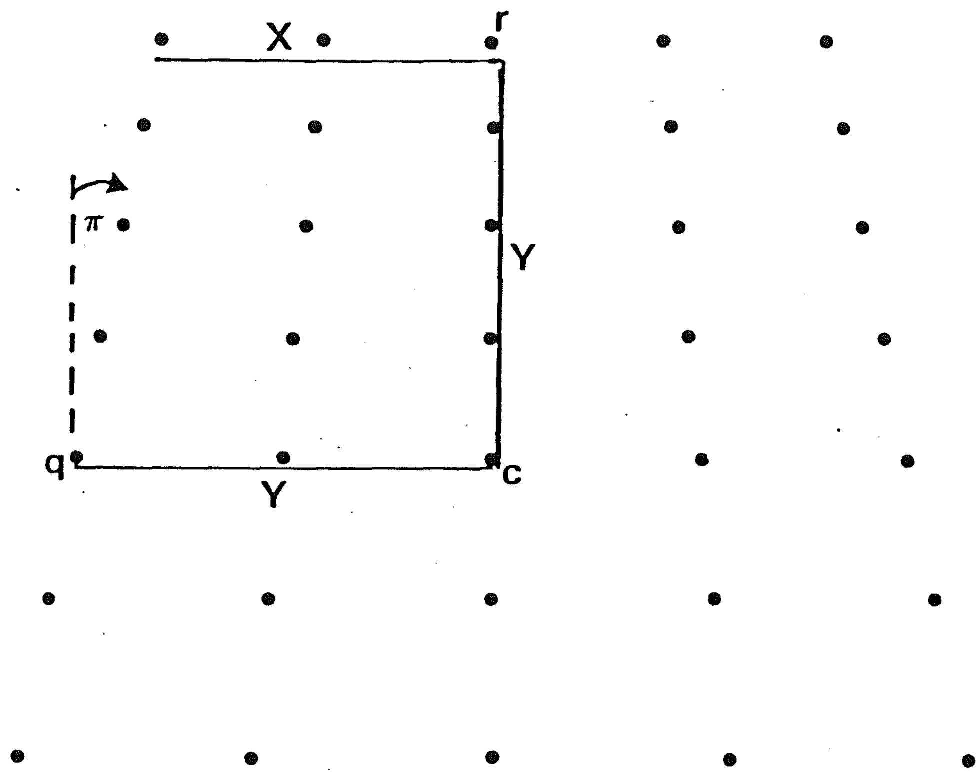



Figure 3. Only two measures are required from the information on a projection plane to extract the angle of slant of the projected surface. cq=cr=Y and cq covers 2 units. Moving across 2 units from r results in a distance X. These values of X and Y can be substituted into equation (6) to obtain the slant angle, theta.

From equation [5]

$$\tan\theta = \frac{\tan\pi}{\tan\delta}$$

Now

$$\tan\pi = \frac{Y-X}{Y}$$

and $\tan\delta = {}^{Y}/_{f}$ where f is the focal length of the optical system.

Therefore

$$\tan\theta = \frac{(Y-X).f}{Y^2}$$

or

$$\theta = \tan^{-1} \frac{(Y-X)}{Y^2} .f \qquad [6]$$

Thus the slant angle θ can be obtained from just two distances on a projection plane. This projection plane could be a photo-sensitive surface such as the retina or a special electronic photo-sensitive device coupled to a camera and lens system.

In figure 3, f is equal to 26cm, Y = 4.6cm and X = 3.6cm. Therefore $\theta = 50°$ for this particular surface.

An optical device could quite readily be built which incorporates these principles and which could produce an output regarding the slant of a static surface relative to its 'line of sight'. This is assuming that textural units on the surface are sufficiently well-defined to be discernable by the system.

This analysis can be criticised from the point that it is limited to static situations and does not seem to incorporate the complex transformations and changes over time which form the basis of our everyday perceptions. However, the human observer can perceive the slant of a static surface and this thesis is concerned with this situation. Besides it would not be difficult to modify the optical device described above, to be sensitive to motion perspective and velocity gradients.

This is not to claim that the human observer extracts slant information from his retinal image by the exact process described above. The analysis simply shows that it is possible to obtain the slant angle $\theta$, from the two-dimensional information on a projection plane and it points out some of the important variables involved in such a process. The analysis forms a useful model for the perception of slant, because factors which result in the model producing 'underestimations' of slant, may also be contributing to the misperception of slant by human observers. The question that requires answering is that if the relatively simple optical device described above, can produce accurate 'monocular static slant judgements', then why does the human observer fail to produce the same degree of accuracy and often make gross perceptual errors?

# CHAPTER III

## A MODEL OF SLANT UNDERESTIMATION

It is possible to begin to answer the question posed at the end of the last chapter by comparing the general slant perception situation with the slant perception experiment situation. In the typical experiment on slant perception an isolated featureless rectangle is viewed against a black or uniform background. In another common type of experiment, the observer views either a photograph or a real surface at a slant, through an aperture in a reduction screen. This arrangement of screen and aperture is designed to eliminate 'extraneous cues' such as outline perspective. In other words only a small part of the total possible optic array is visible to the observer.

Now one difference between the way we normally perceive slant and the situation adopted in slant perception experiments is that the information is presented in a fashion not normally encountered in the everyday environment. We are rarely called on to judge the slant of a surface that rotates backwards or forwards about an axis passing through the point of fixation. By contrast we continually experience changes to the angle between our line of sight and the surface we are standing on or fixed surfaces surrounding us such as walls or ceilings. There is an important difference between these situations and the case of a 'pivoting' surface along our line of sight. In the pivoting case, the distance from the eye to the fixation point on the surface remains constant, but the direction of the perpendicular from the eye to the surface changes with changes to the slant of the surface. This is the type of artificial situation that has typically been utilized in slant perception experiments. What it fails to imitate is the normal visual experience in which the eye remains a fixed distance from a solid surface while the distance from the eye to the fixation point and the direction of the line of regard is constantly changing. One of the significant aspects of slant perception experiments is however, the fact that the reference axes of the experimental room are not visible to the observer while the slant judgement is made.

## 3.1 Deviation of the Apparent Straight-ahead Direction

In the analysis outlined in Chapter II it is assumed that the observer's straight-ahead direction, i.e. the direction perpendicular to the frontoparallel plane, passes through the central pivot axis of the surface. The line from the eye through the centre of the surface is considered by the experimenter, to be the observer's straight-ahead direction. It is the direction perpendicular to the frontoparallel plane from which the angle of slant is measured. The apparatus is aligned such that judgements are made on a response device which has its zero direction perpendicular to this straight-ahead direction.

In the typical slant perception experiment, simultaneous viewing by the observer of the test surface and the external reference system of the room is rare. It is possible therefore that the observer's perceived straight-ahead direction does not coincide with the true straight-ahead. Evidence of a similar effect can be found in studies on eye-level, in which the observer's perception of the direction of the ground plane (eye-level) often deviates greatly from true eye-level when an impoverished visual field is used. This effect also occurs when the field is structured in some particular way and it occurs even though so called postural cues are still present (MacDougall, 1903; Sharp, 1934; Kleinhans, 1970; Perrone, 1977). Such effects have also been noted for the direction parallel to the medial plane (Wapner et al, 1953; Bruell and Albee, 1955). Rock (1975, p.169) suggested "it is as if the perceptual system confuses what is straight ahead, and therefore what is in a direction perpendicular to the frontal plane of the observer's body, with what is perpendicular with respect to a plane at which the observer is looking".

MacDougall (1903) attempted to explain deviation of eye-level in terms of the depression of the line of sight associated with convergence of the eyes. However such an explanation cannot explain the case of deviation from the medial plane and more importantly, it confuses direction of eye-gaze with perceived straight-ahead. The gaze-line can wander even though the straight-ahead direction is fixed, and the two should not be confused. The problem concerns deviation of the apparent straight-ahead and not deviation of the gaze-line.

Consider a line passing through the centre of the eye and running perpendicular to the frontoparallel plane. Call the length of this line, from the eye to where it meets the surface, d. Now consider what happens to the value of d as the head is orientated at various angles in relation to the surface. Only when we are orientated 'straight on' to the surface will d be at a minimum. In other words we are orientated 'straight on' to a plane surface whenever the length of the perpendicular is at a minimum. Under this condition the perpendicular to the frontoparallel plane of the observer is parallel to the perpendicular to the plane of the surface. Notice that this does not presume that the eyes are looking straight ahead in the direction of the perpendicular. I am only using the eyes as a convenient reference point on the face. The centre of the forehead or even the tip of the nose is just as relevant in that when the fronto-parallel plane of the observer is parallel to the surface, the length of the perpendicular to the frontoparallel plane, from that particular part of the face to the surface, is at a minimum.

I propose that under reduction conditions, the observer considers the shortest distance from his eyes to the surface to represent the straight-ahead direction, in accordance with the conditions that normally exist in the environment. For a surface parallel to the observer's frontoparallel plane, i.e. with zero slant, the straight-ahead direction passes through the central pivot axis of the surface because that represents the shortest distance to the surface and so the situation is in keeping with the experimenter's expectations. However, if the surface is now slanted relative to the observer's frontoparallel plane, by pivoting it about its central axis, then the shortest distance to the surface is no longer the distance from the eye to the central axis. The shortest distance will be from the eye to one edge of the surface and it depends on the size of the surface and its angle of slant.

Given the reduction conditions that exist the proposal suggests that the observer will consider his straight-ahead direction to be in the direction of the nearest part of the surface. Thus the observer's perceived straight-ahead will deviate from the true straight-ahead in the direction of the normal to the surface as has been shown experimentally (e.g. Kleinhans, 1970). The observer has 'minimized' the distance between himself and the surface and is thus orientated 'straight on' to the surface in the same way that he is orientated 'straight on' to surfaces in his everyday environment. Certainly he is confusing the direction perpendicular to his frontal plane with the direction perpendicular to the surface as Rock (1975) suggested, but under the reduction conditions that exist, and the unusual form of presentation of slant already mentioned at the beginning of this chapter, it is not impossible that the observer resorts to strategies that correspond to common relationships in his visual environment. The proposal offers an ecologically based explanation for the deviation of the apparent straight-ahead.

In the case of isolated rectangles used in many slant perception experiments it is proposed that perceived straight-ahead is taken by the observer to be the direction from the eye to the nearest part of the surface. In the case of aperture experiments, this direction is considered to lie in the direction of a line joining the eye to the edge of the aperture corresponding to the closest part of the surface. This proposal applies to the reduction situation used in slant perception experiments where the reference axis of the experimental room is obscured from the observer. In most cases the proposal suggests that the perceived straight-ahead direction will not coincide with the true straight-ahead.

The fact that perceived straight-ahead may deviate from the true straight-ahead does not immediately imply slant underestimation. An obvious consequence of such an error is however, that the apparent fronto-parallel plane no longer coincides with the true fronto-parallel plane and so judgements made in relation to the apparent position will be in error.

We can develop a general equation for determining the perceived slant of a surface given that the perceived direction of the straight-ahead direction lies in the direction of the shortest distance to the surface. In figure 4, let OQ stand for the perceived straight-ahead direction and OC is the true straight-ahead. $QZ'$, perpendicular to OQ, represents the

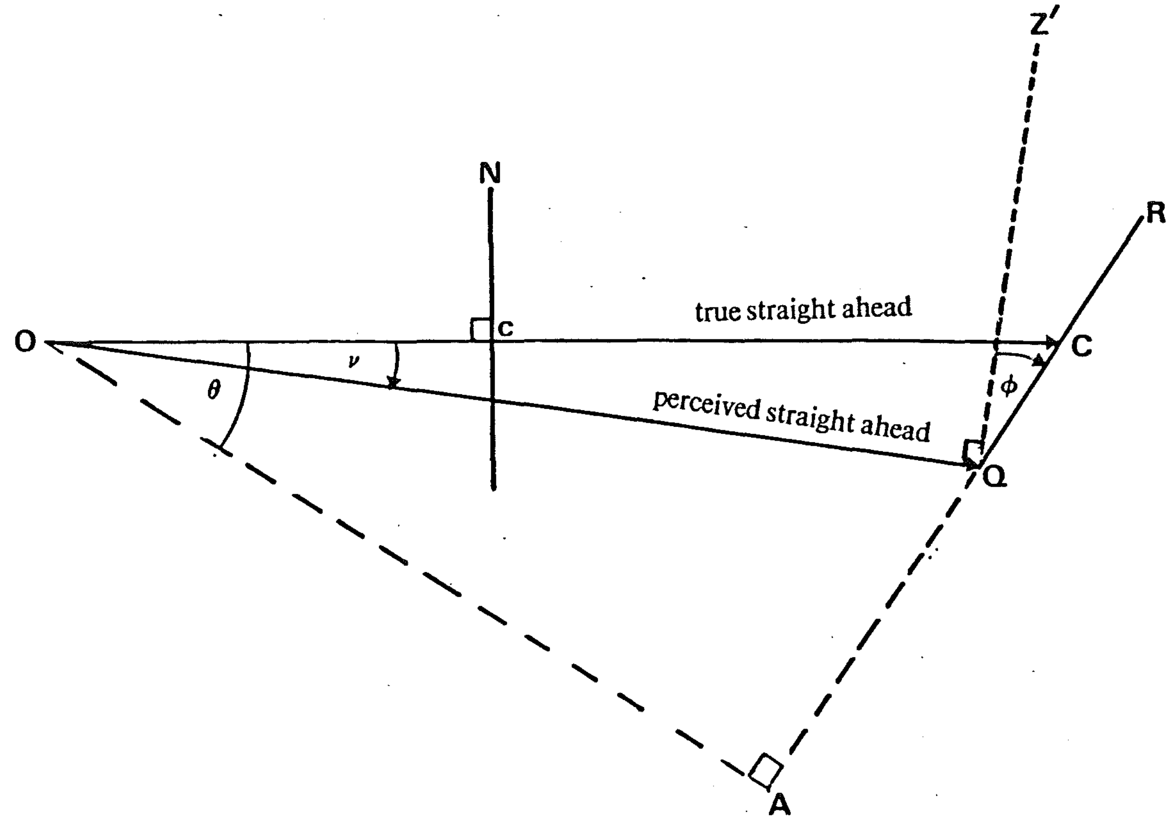



Figure 4. Deviation of the perceived straight ahead in the direction OQ results in the apparent frontal-parallel plane appearing to lie in the direction QZ′. The slant of the surface QR is judged in relation to QZ′ rather than cN and the judgment is in error.

apparent fronto-parallel plane of the observer. cN is the true fronto-parallel plane. Let $\nu$ be the angle between the true and perceived straight ahead directions. It can be shown that the surface QR is rotated back from the apparent fronto-parallel plane QZ' by an angle $\phi$ such that $\phi = \theta - \nu$. In figure 4 $\angle QOA = \angle RQZ'$ and $\angle QOA = \theta - \nu$. Therefore $\phi = \theta - \nu$. This means that if the true slant of the surface is $\theta$, we can expect the slant of the surface to be underestimated by an amount $\nu$.

Given the large amounts of underestimation quoted in Chapter I, it is difficult to see how such a model could account for it, unless $\nu$ is very large. A quick examination of the dimensions of the surfaces used and the viewing distances, reveals that $\nu$ rarely exceeded 10°, and this would not produce the order of underestimation that usually occurs. Also for aperture experiments the prediction is that a constant amount of underestimation should occur over all angles. Again this is something which is not evident from the experimental data. Such a simple direct consequence of the misperception of the straight-ahead direction does not provide an adequate model of slant underestimation. A more complex mechanism will now be proposed.

## 3.2 Main Model of Underestimation

Deviation of the apparent straight-ahead plus incorrect 2-dimensional information.

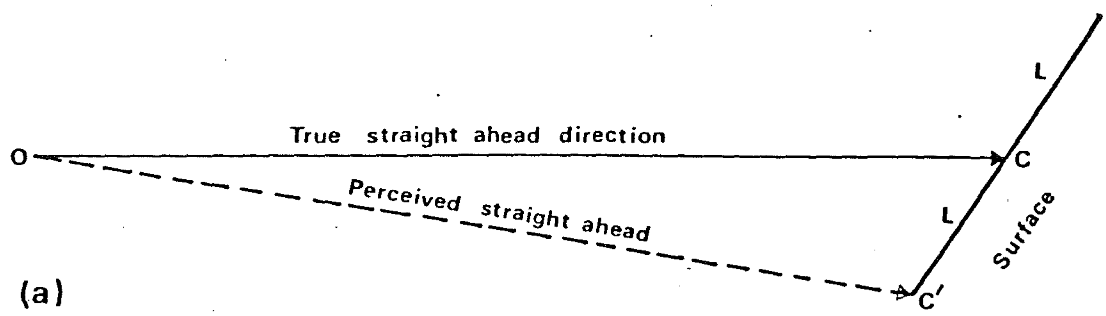
In the analysis outlined in Chapter II, a system was developed for extracting $\theta$ from two-dimensional information. This system was based on two main features; a distance Y related to the projected length of the surface and an angle $\pi$, related to the width of the surface. If one or both of these factors is misperceived, the perceived slant would be in error. This could occur if the perceived straight-ahead direction deviates from the true direction, as outlined in 3.1 above. We begin with the original proposal that the perceived straight-ahead is taken by the observer to be the direction from the eye to the nearest part of the surface, and limit ourselves initially to the case of isolated slanted rectangles.

3.2.1 Slanted Rectangles (see figures 5a, b)

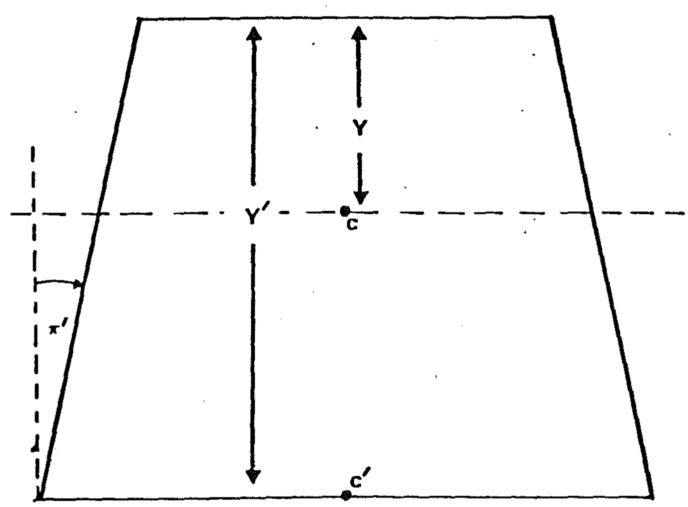
Let Y be the true projected length of half of a surface length 2L, when the straight-ahead direction passes through the midpoint C. Then Y' is the projected length of the surface when the straight-ahead direction is considered to pass through C', and this length is used in the evaluation of $\theta$ instead of Y. [Proposal 1]

In the case of an isolated rectangle, the only converging lines visible to the observer are those projected by the extreme edges of the figure. It is proposed that the convergence angle of these lines is used in the evaluation of $\theta$. The use of this angle is a second perceptual error, because it is based on the assumption that the true shape of the figure is a square. Only when the figure is a square, does the length of the surface represented by Y equal the distance from C to the outer edge, ($^W/_2$). However, the model states that given no other value of $\pi$, the angle of convergence of the outline of the figure ($\pi'$) is used in the evaluation of $\theta$, and this is correctly registered. [Proposal 2]

Proposals 1 and 2 can be summarized as follows: given an isolated slanted rectangle, the projected length of the total figure Y' is used in the derivation of the slant angle $\theta$, instead of the projection of half of the figure, and the angle of convergence of the outlines of the figure, $\pi'$, is used instead of the convergence of lines a distance Y from the centre of the projection plane along the projected axis of rotation. It will be shown that in the majority of cases, such perceptual errors lead to the slant of the figure being underestimated and the amount of underestimation




(a)




(b)

**Figure 5.** (a) The perceived straight ahead coincides with the shortest distance to the surface and hence lies in the direction OC' instead of OC.

(b) When the perceived straight ahead passes through C' (represented by c' on the projection plane) the projected length of 'half' of the surface is Y' instead of Y.

can be calculated given the various experimental parameters. The next step is to derive the equation for the predicted slant estimate $\beta$, when $\pi = \pi'$ and $Y = Y'$.

Consider a surface QR, length 2L, slanted away from the frontal plane (represented by CN in figure 6) through an angle $\theta$. Let the width of the surface be w.

Now from equation [5] in Chapter II,

$$\tan\pi' = \tan\beta.\tan\delta'$$

$\tan\delta'$ represents the total projected length of the figure, and $\tan\delta' = \tan\nu_1 + \tan\nu_2$, where $\nu_1 = \angle COQ$ and $\nu_2 = \angle COR$ in figure 6. Therefore

$$\tan\beta = \frac{\tan\pi'}{\tan\nu_1+\tan\nu_2} \qquad [7]$$

Now, according to proposal 2 of the model, $\tan\pi' = \tan\pi$ therefore from equation [4] in Chapter II,

$$\tan\pi' = \frac{\tan\theta.w}{2D} \qquad [8]$$

In figure 6,

$$\tan\nu_1 = \frac{L.\cos\theta}{D-L\sin\theta} \qquad [9]$$

and

$$\tan\nu_2 = \frac{L.\cos\theta}{D+L.\sin\theta} \qquad [10]$$

Substituting [8], [9] and [10] into equation [7] gives,

$$\left[\quad \beta = \tan^{-1}\left(\frac{w.\sin\theta(D^2-L^2\sin^2\theta)}{4LD^2\cos^2\theta}\right)\quad\right] \qquad [A]$$

This is the perceived slant angle predicted by the model for a rectangular figure distance D from the eye, width w and length 2L at a true slant angle of $\theta$, when the perceptual errors outlined in proposals 1 and 2 occur.

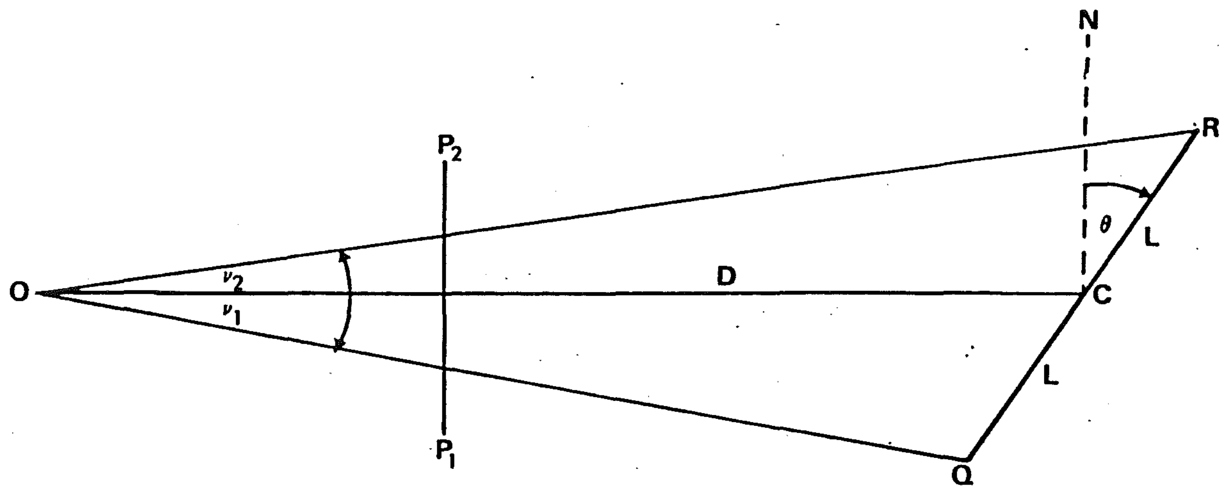



Figure 6. Representation of the geometry involved for a surface length 2L and width W, slanted at an angle theta and viewed from a distance D. The information on a projection plane $P_1P_2$, an arbitrary distance f from O, is used in the evaluation of theta.

## 3.3 Aperture Presentation

For aperture presentations with textured surfaces, the general principle of the model is the same, but consideration must be given to the fact that only a limited part of the surface is visible to the observer, and that texture replaces outline information. It is assumed that in the majority of texture patterns some 'linearity' or perspective information is discernable. Highly irregular or very fine textures would make evaluation of the angle of convergence difficult and such a situation would not be expected to comply very well to the model. Besides, such stimuli have been shown to result in very poor slant estimates (Braunstein, 1968).

As in the case of isolated rectangles, we begin with the assumption that the observer considers the nearest part of the surface to be the direction of the straight-ahead. In the majority of cases, this coincides with one edge of the viewing aperture. On some occasions however, if the aperture is large enough, the nearest part of the surface may lie within the field of view of the aperture. Irrespective of what situation actually exists, the model still states that the total visible projected length of the surface is used in the evaluation of $\theta$.

In the case of an isolated rectangle, L as used in the model, refers to half of the length of the rectangle and since the rectangle pivots about its central axis, either the length of the nearest or furthest half could be used since they are both equal. However, when an aperture is used, the balance of distances to either side of the fixation point, is no longer equal (see figure 7). $L_1$ is not equal to $L_2$ as was the case for isolated rectangles.

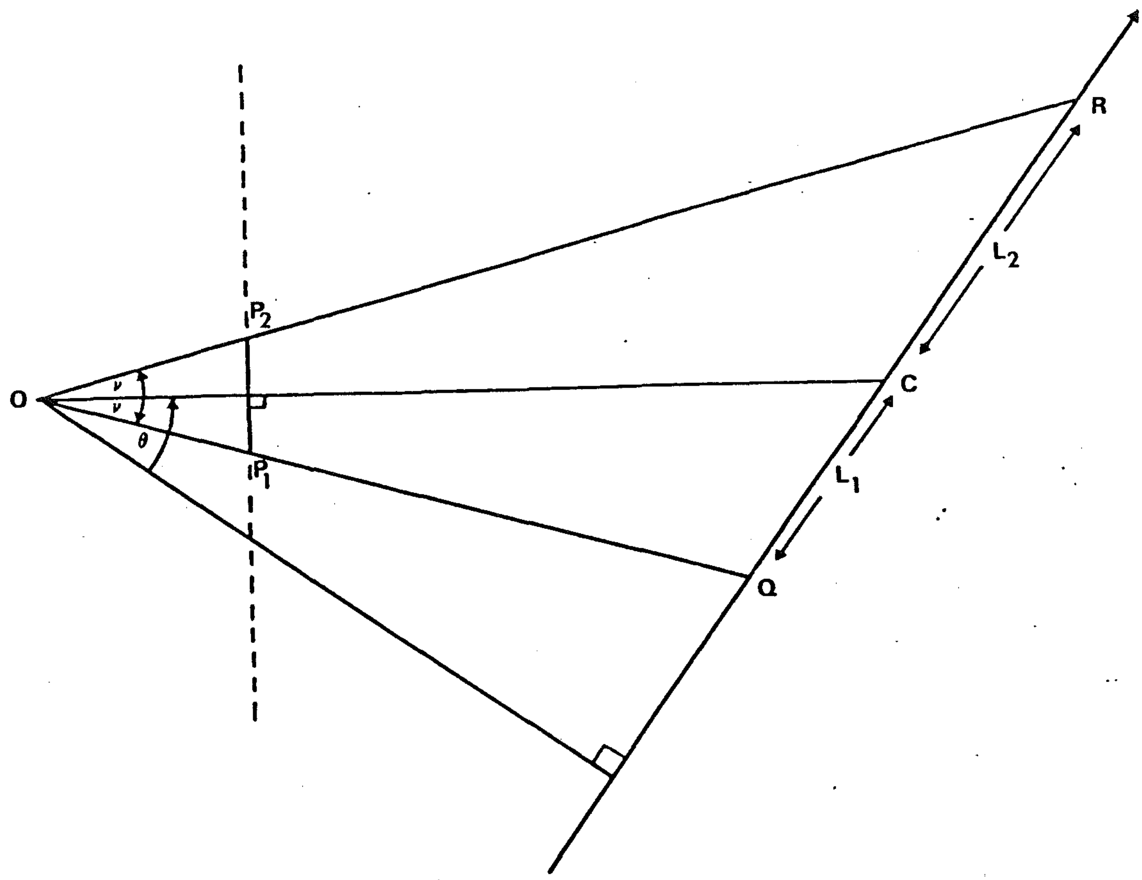



Figure 7. When a surface QR is viewed through an aperture $P_1P_2$ the extents of the surface on either side of the centre C, are not equal (unless theta = 0°). The projected length of the visible surface subtends an angle at the eye equal to the field of view of the aperture.

The projected length of the surface Y' is represented by $\tan\delta'$ such that $\tan\delta' = 2\tan\nu$ where $\nu$ is half of the field of view of the viewing aperture or more specifically the field of view of the aperture in the direction perpendicular to the axis of rotation of the surface.

Therefore from equation [5]

$$\tan\beta = \frac{\tan\pi'}{2\tan\nu}.$$

and

$$\tan\pi' = \frac{\tan\theta . w}{2D} \qquad \text{(from equation 8)}$$

Therefore

$$\beta = \tan^{-1}\left(\frac{\tan\theta . w}{4D . \tan\nu}\right) \qquad [B]$$

This is the equation for the estimated slant angle predicted by the model for a surface viewed through an aperture with a field of view = $2\nu$.

The problem arises when one must consider which dimensions of the visible surface are to be used for the value of w. The observer could simply use the width of an individual unit on the surface, but such a strategy would necessarily be sub-optimal. We can define the upper limit on w as the maximum width of the surface visible through the aperture. Two different configurations need to be considered.

### 3.3.1 Circular Apertures

The maximum width of a surface visible through a circular aperture, occurs at the diameter parallel to the axis of rotation of the surface. This is for the usual case of an aperture centered at the axis of rotation of the surface. As well as this the surface must always completely fill the field of view of the aperture, otherwise this property does not hold.

Therefore, for a circular aperture, we can replace w in equation [B] with $2D\tan\nu$. This is the visible width of the surface at distance D from O, when viewed through an aperture with a field of view at the diameter equal to $2\nu$.

This gives as an upper limit to the predicted value of $\beta$ (the slant estimate):

$$\beta_{max} = \tan^{-1}\left(\frac{\tan\theta}{2}\right) \qquad \text{(circular apertures)}$$

Thus a circular aperture has the unique and interesting property that the maximum slant estimates as predicted by the model, can never exceed $\tan^{-1}\left(\frac{\tan\theta}{2}\right)$ irrespective of the size of the aperture. For instance, if the surface was at a true slant of 50°, we would expect the predicted slant estimates to never exceed 30.8° no matter how large or small we made the aperture.

### 3.3.2 Rectangular Apertures

For a rectangular aperture, the largest width of the surface visible through the aperture is that subtended by the edge of the aperture in line with the most distant part of the surface, D'.

Let $\lambda$ be half of the field of view of the aperture in the direction parallel to the axis of rotation of the surface.

Then w is closely approximated by $w = 2\tan\lambda . D'$.

Now

$$D' = \frac{D\cos\theta}{\cos(\theta+\nu)}$$

Therefore

$$w = \frac{2\tan\lambda . D\cos\theta}{\cos(\theta+\nu)} \qquad [11]$$

Substituting this into equation [B] gives

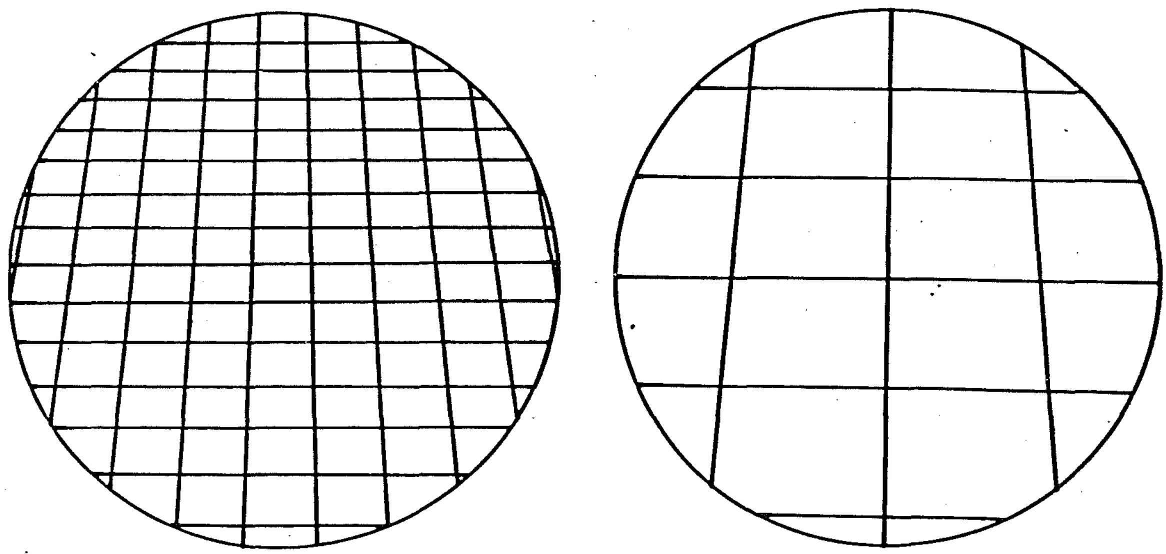
$$\beta_{max} = \tan^{-1}\left(\frac{\tan\lambda . \sin\theta}{2\tan\nu . \cos(\theta+\nu)}\right) \qquad \text{(rectangular apertures)}$$

where $\nu$ is half of the field of view of the aperture in the direction perpendicular to the axis of rotation of the surface.

## 3.4 Specific Predictions for Aperture Presentations

The maximum values predicted by the model in 1 and 2 above would only be attained by an observer if perspective lines are visible at the extreme edge of the aperture. This may or may not be the case depending on the size and spacing of the elements on the surface (see figures 8a, b). In figure 8a there are perspective lines at the outermost edge of the surface close to the edge of the aperture and the predicted slant estimate would be close to the maximum as defined in the equation for $\beta$ (max) above. However, in figure 8b, w corresponds to the width of the surface between the outermost two lines but this is less than the maximum value defined in 3.3.1 above.

Computer programs were developed (see Appendix A) based on an iterative process, which calculated the width of the surface between the outermost perspective lines just included within the field of view of the aperture. The values for $\nu$, $\lambda$, $\theta$ and D are input into the programs along with the width of the smallest unit on the surface (this could also be the mean spacing between dots in the case of a random-dot surface). The resulting value of w is substituted into equation [B] to obtain the predicted slant estimates for the particular aperture and surface combinations used.



(a) (b)

**Figure 8.** (a) Perspective lines are visible at the outer edges of the aperture and so the predicted slant estimate in this situation would be close to the maximum value given by the equation for beta (max.); (b) The outermost perspective lines include a width w which is less than the full width of the surface visible through the aperture. The predicted slant estimate will be less than that given by the equation for beta (max.)

In summary, the model proposes that the direction perceived to be straight-ahead by the observer, deviates in the direction of the nearest part of the surface. In the majority of cases this will not coincide with the true straight-ahead direction. This deviation is dependent upon the fact that 'reduction' conditions exist with the reference axis of the room obscured from the observer. Given that this deviation occurs, the model then proposes that the total projected length of the surface is used in the evaluation of the slant angle, along with the angle of convergence of the outermost edges. Both of these factors constitute a perceptual error in the majority of cases, and evaluation of the slant angle on this basis will result in a predicted slant estimate equal to $\beta$, given by equation [A]. It will be shown that $\beta$ is usually less than $\theta$, the true slant angle, so the model predicts slant underestimation. Equations were developed for predicting the slant estimates expected for both isolated rectangle test surfaces and for aperture presentations. As long as the various experimental parameters are available, it is possible to assess the model in relation to some of the past experiments discussed in Chapter I, and this will be carried out in the next chapter.

# CHAPTER IV

# ASSESSING THE MODEL

## 4.1 Features of the Model

By examining how changes to the various parameters of the model affect the value of $\beta$, we can make an assessment of the model in relation to some of the variables tested in past experiments.

### 4.1.1 Changes to the Size of a Rectangular Test Surface

Examination of equation [A] shows that a decrease in L should result in an increase in the slant judgements, and an increase in w will also produce an increase. In other words, the larger the ratio $^W/_L$ the greater the slant judgement. A short wide rectangle should produce greater slant judgements for a given value of $\theta$, than a tall narrow rectangle. Notice that we need to consider both the width and height of the test rectangle under the conditions of the model. Flock (1965) in an attempted assessment of the variables affecting slant judgements across many studies, only considered the angular height of the rectangles (2L) and noted that from the data, it appeared that the smaller the rectangle, the better the slant judgements became. This is in keeping with the model, but it is confounded evidence since it ignores the width of the rectangles.

A better evaluation is obtained by comparing Smith's (1956) study with the Smith (1964) study. In the former, a rectangle measuring 28cm x 15.4cm was used ($^W/_L$ = .91) and in the latter a rectangle measuring 25.4cm x 26.32cm was used ($^W/_L$ = 1.6). Flock (1965) derived mean regression coefficients from Smith's 1956 and 1964 data of .41 and .68 respectively. This supports the prediction of the model that the higher the $^W/_L$ ratio the better the slant judgements.

Apparently contrary evidence against the model comes from Stavrianos (1945) and Freeman (1966a). Stavrianos (1945) reported that of two plane rectangular stimuli presented at the same slant, the larger is perceived as being at a greater slant. This was the case even when the viewing conditions were unrestricted and binocular. Freeman (1966a) duplicated Stavrianos's findings. The $^{W}/_{L}$ ratio of the two rectangles was identical so at first sight this evidence seems contrary to the predictions of the model. But, in these studies, the two rectangles are presented side by side in the observer's field of view and a comparative judgement is made. Such a situation prevents the mechanism outlined in the model from occurring, because the shortest distance to the surface, and consequently the perceived straight-ahead direction, can no longer be equated with the nearest edge of the surface. In such a situation, the larger rectangle would possess the greatest angle of convergence, as pointed out by Freeman (1966a) and would consequently be judged to be at the greatest slant. It is interesting to note that in a follow-up study by Freeman (1966c), he attempted to avoid the possibility of observers making a 'retinal match' which could occur when the two rectangles are presented side by side (Flock, 1965). This time Freeman (1966c) presented the standard rectangle and the comparison rectangle intermittently by using two chambers of a tachistoscope. Each rectangle was therefore viewed separately. In this situation we would expect the model to be applicable and we note that under these conditions Freeman (1966c) found that the inter-subject variability was high and that the results as a function of size 'were not clearcut'.

### 4.1.2 Changes to the Viewing Distance D

Since D occurs in both the numerator and denomenator of equation [A] the model predicts no changes to the slant estimates with changes to D, when all the other variables are held constant. The only study which manipulated the distance of the test surface was carried out by Gruber and Clark (1956). However the surface was textured and viewed through an aperture and is not appropriate for this particular test of the model, since changing D also changes the values of L and w in equation [A].

### 4.1.3 Apparent Position of the Fronto-parallel Plane and the Viewing Aperture

If the direction of the apparent straight-ahead position deviates from the true position then the position of the apparent frontoparallel plane deviates from the true position. This means that if a reduction screen is being used, the apparent position of the aperture does not coincide with the true position (see figure 9). If OC' deviated downwards for instance, then $P_1P_2$ would be perceived as lying in the direction $P_1' P_2'$ (90° to OC').

If such a state of affairs did occur then one would expect some form of distortion to appear. If, as for example in the case portrayed in figure 9, the apparent position of the top edge of the aperture $P_2'$ was further from the eye than its true position, then the width of the aperture at $P_2'$ would need to be wider than at $P_2$, if it was to subtend the same angle at the eye, while lying a greater distance away. For a square aperture one would expect the aperture to be distorted and appear as a trapezium.

Gruber and Clark (1956) noted an 'interesting effect' in regard to the reduction screen used in their study. When a textured surface was viewed through a square window with the view filling the window, the

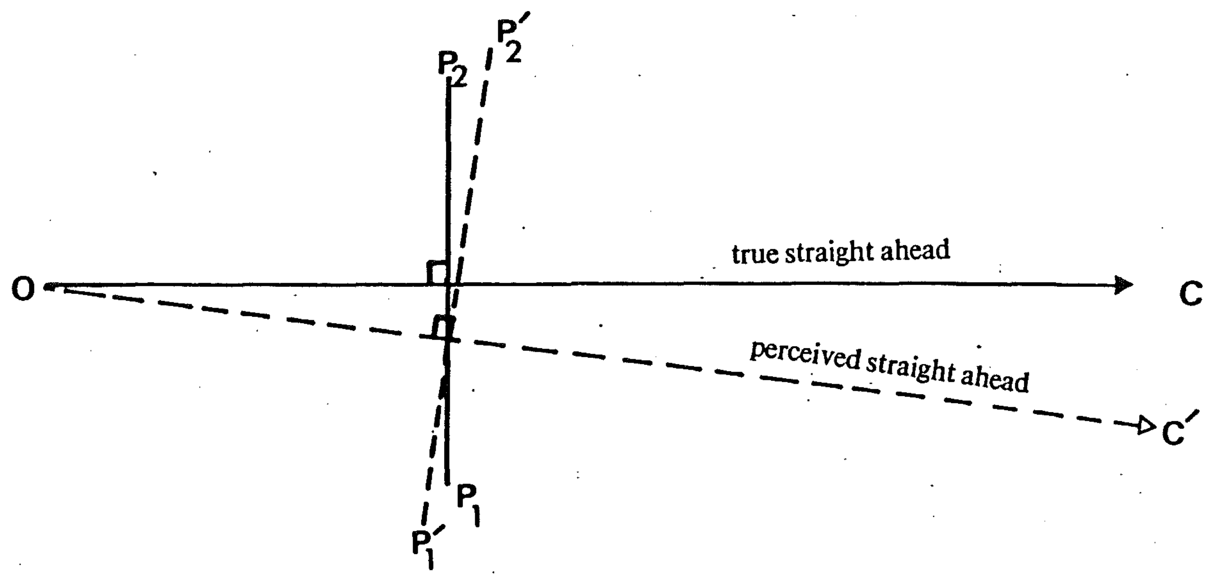



Figure 9. The apparent straight ahead direction deviates in the direction of OC′. The apparent position of the aperture, $P_1'P_2'$, does not coincide with the true position $P_1P_2$. Since $P_2'$ is further from the eye than $P_2$ and $P_1'$ is closer than $P_1$, the shape of the aperture will appear distorted.

apparent shape of the window was distorted and appeared as a trapezium. They minimized this distortion by surrounding the window with a prominent border and by using a circular aperture. Gruber and Clark (1956) noted that the apparent shape of the trapezium was such that the long side was at the end corresponding to the more distant part of the surface. This is in exact accord with the prediction made above. The model can therefore provide an explanation for a hitherto unexplained geometric illusion*.

## 4.2 Testing the Model Against Existing Data

### 4.2.1 Isolated Rectangles

The predictions of the model will now be compared with data from four slant perception studies, which used isolated rectangles as the test surface. These four studies have been selected because full information was provided regarding the experimental situation and a range of slant angles were used.

a) Clark, Smith and Rabe (1955): As part of a study testing the effectiveness of retinal gradients of outline convergence on the perception of slant, Clark et al (1955) used a rectangle measuring 28cm x 15.4cm at a distance of 163cm from the observer's eye. This surface was set at various angles of slant about a vertical axis. In terms of the model we have $D = 163$, $L = 14$, $w = 15.4$. Values for $\theta$ were 0°, 20° and 40°. The Clark et al data was based on the mean of 28 observations. The above values of D, L, w and $\theta$ are substituted into equation [A] to produce the predicted values of the slant estimates. These, along with the Clark et al (1955) data are presented in Table 1 and plotted in figure 10a.

The values of $\beta$ and the experimental data are in close agreement.

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*This illusion can also be seen in two-dimensional displays, e.g. see figure 15 and it seems to parallel Ehrenfel's (1890) variant of the Ponzo illusion.

b) Smith (1956): Smith carried out a follow up study to the Clark et al (1955) study designed to determine differences in perceptual error for two kinds of forms. His stimulus condition A used a white rectangle with the same dimensions as that used by Clark et al (1955). The distance D was again 163cm. but a wider range of slant angles was tested. Each mean was based on 32 observations. The values of $\beta$ predicted by the model and Smith's (1956) data are presented in Table 1 and plotted in figure 10b. Again the model is in good agreement with the experimental data.

c) Smith (1959): In another study involving outline convergence Smith included a condition which used a textureless rectangle measuring 27.94cm x 15.24cm. The distance from O to the surface was reported as being 173cm. Each mean was based on 33 observations. Table 1 presents values of $\beta$ and Smith's (1959) data and they are plotted in figure 10c. Close agreement between the values of $\beta$ and Smith's data is evident.

d) Smith (1966): Smith carried out a study designed to investigate the effect of shape constancy on phenomenal slant. D was equal to 228.6cm and a rectangle measuring 25.4cm x 20.32cm was used. Each mean was based on 72 observations and three slant angles were tested; 10°, 30° and 40°. Values of $\beta$ and Smith's data are shown in Table 1 and plotted in figure 10d. There is close agreement between data and model.

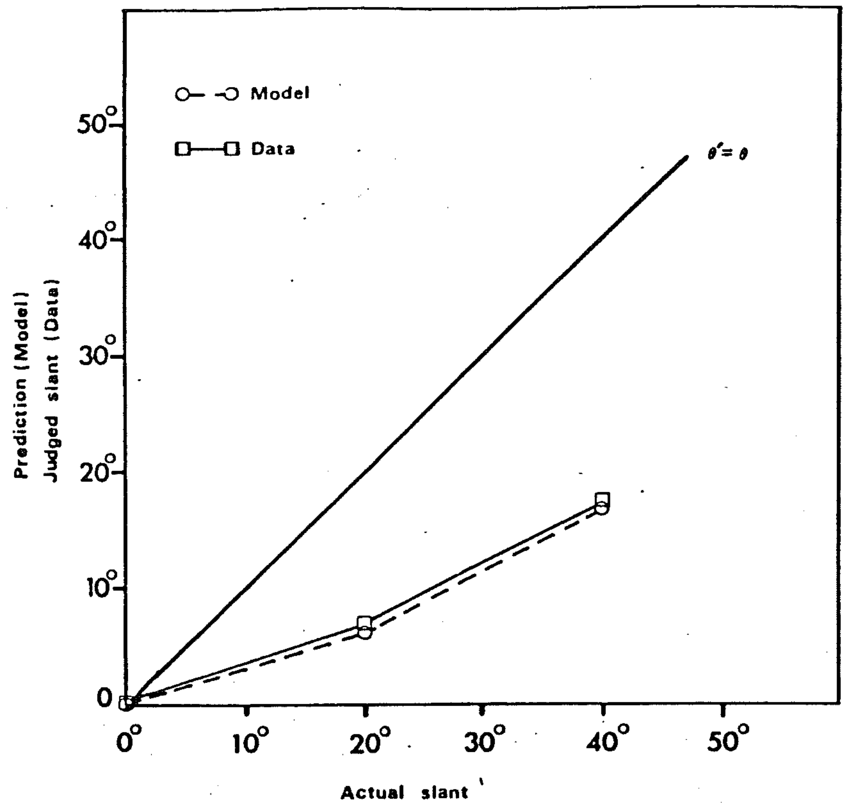
Sufficiently different conditions have been used across the four studies tested to support the general nature of the model. It is felt that the large number of subjects tested in these studies is sufficient to warrant not carrying out further experiments involving isolated rectangles. Sufficient data exists showing that the model can account for the large amount of underestimation that typically occurs in these experiments.

TABLE 1

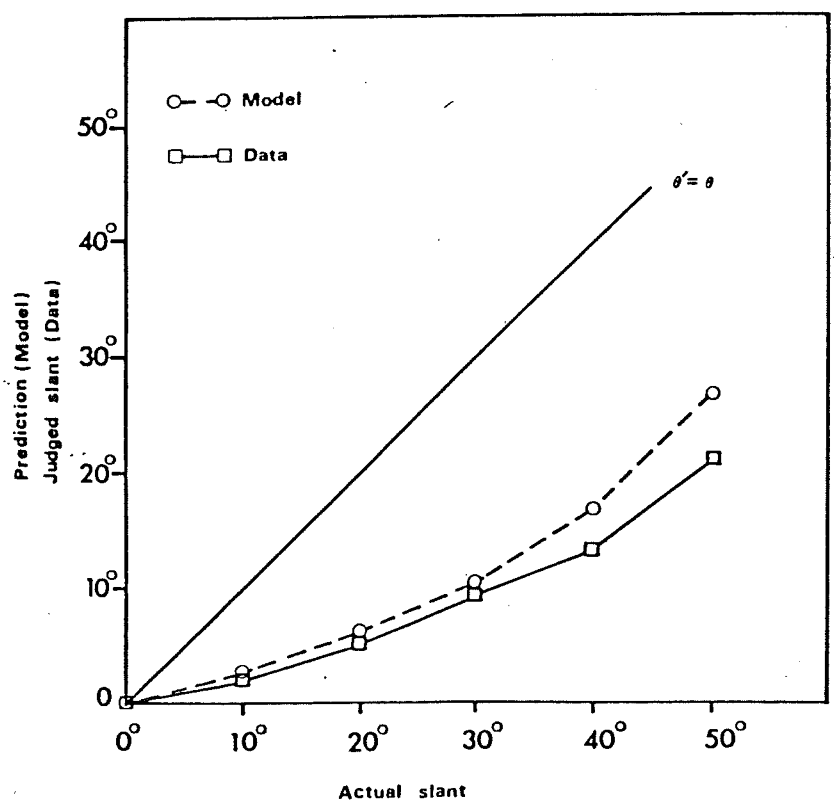
| Experiment | D | L | w | θ | β | Data |
|---|---|---|---|---|---|---|
| | (centimeters) | | | (Degrees) | | |
| Clark et al (1955) | 163 | 14 | 15.4 | 0 | 0 | .14 |
| | | | | 20 | 6.08 | 6.96 |
| | | | | 40 | 16.72 | 16.96 |
| Smith (1956) | 163 | 14 | 15.4 | 0 | 0 | .09 |
| | | | | 10 | 2.82 | 2.06 |
| | | | | 20 | 6.08 | 5.13 |
| | | | | 30 | 10.37 | 9.77 |
| | | | | 40 | 16.72 | 13.29 |
| | | | | 50 | 26.91 | 21.00 |
| Smith (1959) | 173 | 13.97 | 15.24 | 0 | 0 | -.79 |
| | | | | 10 | 2.80 | 2.39 |
| | | | | 25 | 7.99 | 7.03 |
| | | | | 40 | 16.59 | 13.47 |
| Smith (1966) | 228.6 | 12.7 | 20.32 | 10 | 4.10 | 3.3 |
| | | | | 25 | 11.62 | 9.5 |
| | | | | 40 | 23.63 | 23.1 |

Table 1: Testing the model against data from four existing studies.

D = distance from eye to surface
L = half of the length of the surface
w = width of the surface
θ = actual slant of the surface tested
β = slant estimates predicted by the model from equation [A]
Data = mean slant estimates from experiments

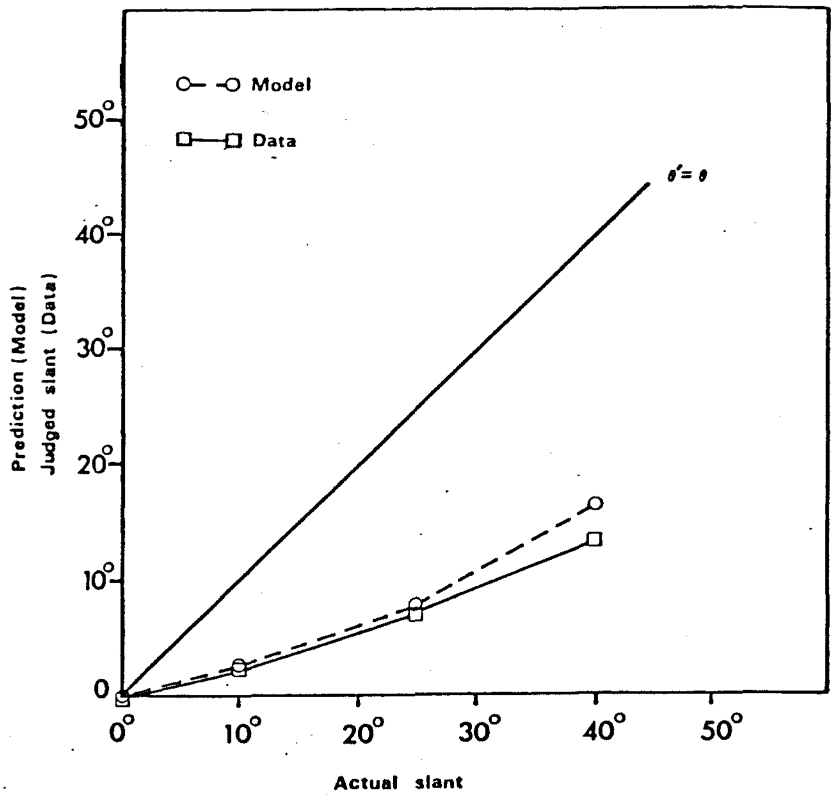



(a)

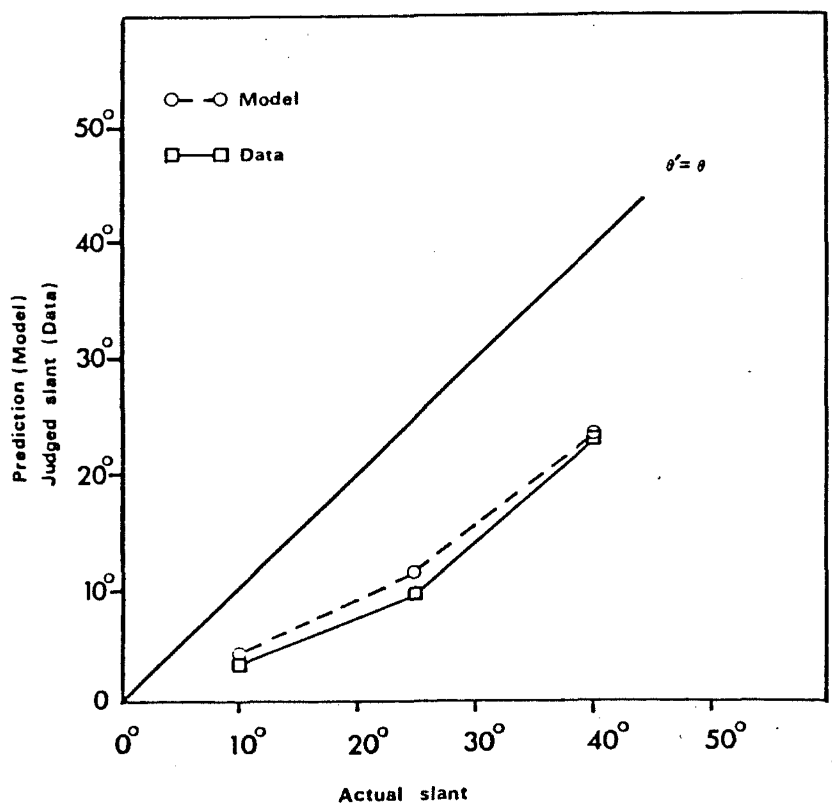



(b)

**Figure 10.** Comparison of existing experimental data with the predictions of the model. The line $\theta' = \theta$ represents veridical slant judgments and when compared with the data it reveals the extent of the underestimation that occurs in these studies.
(a) Clark, Smith and Rabe (1955).
(b) Smith (1956)




(c)




(d)

Figure 10. (c) Smith (1959)
(d) Smith (1966)

4.2.2 Aperture Experiments

Unfortunately very few studies have been carried out in the field of slant perception which used a regularly textured surface viewed through an aperture. Gibson (1950a) used a regular bricklike wallpaper pattern for one of his conditions, but no details were reported regarding the size of the units in the pattern nor the distance from the camera to the surface, used for preparing the photographic test slides. Flock and Moscatelli (1964) included several texture patterns which could be considered regular, but their results were reported in terms of a combined regression coefficient over nine separate angles.

The remaining slant experiments using apertures have used varying degrees of irregularly textured surfaces. As it stands, the model cannot be readily applied to cases of irregularly textured surfaces without some assumption being made as to how the observer extracts the 'linear perspective' information from the irregular distribution of elements on the surface. Only when the simplest case of regularly distributed elements has been assessed can we attempt to extend the model to the irregular case.

In light of the lack of empirical data for assessing the model in the aperture situation, it was decided to carry out a series of experiments to test this aspect of the model.

# CHAPTER V

# EXPERIMENT 1

## 5.1 Introduction

The model predicts the estimated slant in the case of aperture presentation to be given by equation [B], i.e.:

$$\beta = \tan^{-1}\left(\frac{\tan\theta . w}{4D\tan\nu}\right)$$

The value that is used for w depends on the shape of the aperture and the size of the elements on the surface.

The simplest case of a circular aperture was chosen for the initial experiments and the surface consisted of a pattern of regular square units. A reliable method of preparing and presenting the test surfaces was required. Back-projected photographic transparencies as used by Gibson (1950a) provide the greatest versatility as far as the quick presentation of a variety of surfaces and angles is concerned, but the photographic system produces artifacts such as an uneven depth of field. The idea of two-dimensional presentations was retained, but the surfaces were generated by a specially designed computer-graphics program developed by the author (see Appendix B).

This program is based on the projective rules outlined by Kubert, Szabo and Giulieri (1968) and it carries out a perspective transformation on points lying in 3-dimensional space. With this program a surface can be defined anywhere in 3-dimensional space in terms of a regular grid pattern of points and when parameters such as the distance from the surface to the eye or the angle of slant are input, the correct 2-dimensional perspective representation is produced on a 'Calcomp' plotter (see figure 11a, for example).

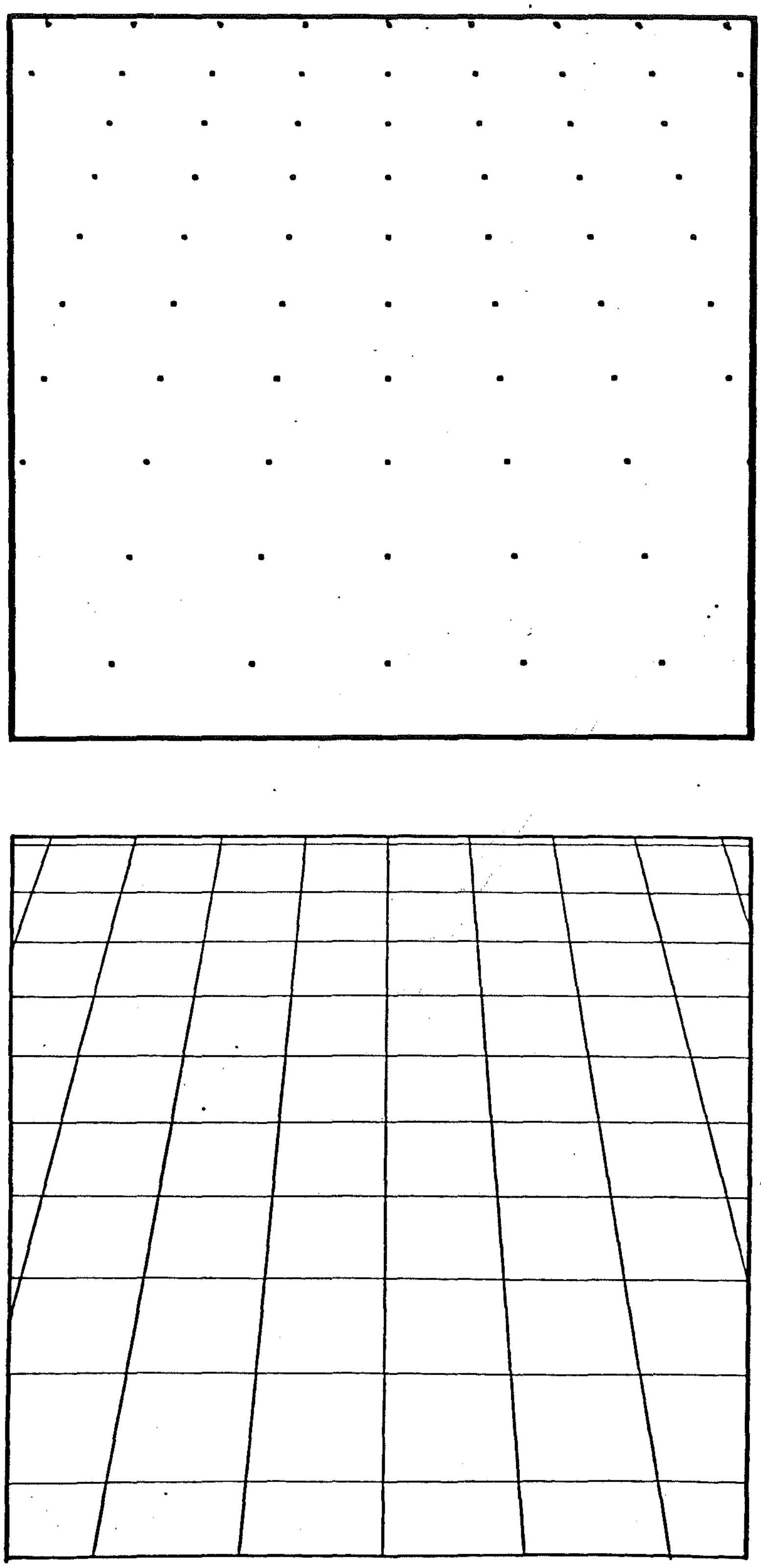
The points on the final plot were traced and joined by straight lines to produce a grid surface made up of regular squares (see figure 11b).

These drawings were then photographed onto 35mm negative film and projected as negatives through a red filter to produce red lines against a black background. The final result is comparable to Gibson's (1950a) system of photographing a real surface at various angles of slant, but the plotting system gives greater control over factors such as field of view and magnification, while removing the problem of depth of field. It also enables complete control over the variables in equation [B].

The slides are back projected onto a transluscent screen a set distance from the observer's eye and the magnification of the projected image is determined such that the information reaching the observer's eye is exactly the same as would be produced by a real surface at the particular distance and slant of the 'artificial' surface.

One of the functions of the first experiment was to assess the authenticity of such a system of presenting the test surfaces and this was achieved by comparing the results from the picture system with those obtained using a real surface.

The real surface was located at a distance of 95cm from the subject's eye and each unit on the surface was a square measuring 8.5cm x 8.5cm.



(a)

(b)

Figure 11. (a) Calcomp plotter output from perspective program. (Appendix B); (b) Example of stimulus surface drawn from (a). The outside frame is never visible through the aperture.

Equivalent picture surfaces were prepared by inserting these parameters into the perspective program (Appendix B) and these were projected in such a way as to produce an equivalent configuration to the real surface.

A circular aperture with a 24° field of view was used. When the above dimensions are applied to the program for circular apertures (Appendix A), we obtain the following predicted values for the slant estimates, given the actual slant values $\theta$:

| | | | | | | |
|---|---|---|---|---|---|---|
| $\theta$ | 0° | 10° | 20° | 30° | 40° | 50° |
| $\beta$ | 0° | 4.24° | 8.71° | 13.66° | 19.45° | 26.64° |

The first hypothesis was that slant judgements for the real surfaces will not be significantly different from slant judgements for the picture surfaces. If a difference in slant estimates is found, then the continued use of picture surfaces for further experiments is not justified.

The second hypothesis related to the predictions of the model and states that the slant estimates will lie close to the values of $\beta$ given above.

The model should be applicable to surfaces slanted either backwards or forwards as well as surfaces pivoted about a vertical axis. There do exist differences in our environment concerning the distribution of surfaces in our visual field. For instance ground planes are more common than surfaces above our eye-level. Whether or not such differences will produce different results in the case of backward slant from those of forward slant judgements is not immediately obvious. The experiment was also designed to test for any such differences.

## 5.2 Method

### Subjects

The subjects were 10 students from the University of Canterbury, all volunteers. They consisted of 6 males and 4 females.

### Apparatus

The apparatus consisted of two parts: the eye-hole/shutter and aperture system and the test surface section.

The subject viewed the test surfaces monocularly with his/her preferred eye, by looking through a Lafayette shutter (aperture fully open) mounted in the front face of a hardboard chamber measuring 100cm wide x 60cm high x 50cm deep. The rear face of this chamber was fitted with a cardboard reduction screen in which an aperture was cut out. The distance from the eye to the aperture was 53cm and the aperture was 22cm in diameter, giving a 24° field of view of the surface behind it.

A chin and head rest ensured that the subject was located at the correct position when viewing the surface. The chin rest was also fitted with a small spring-loaded switch, wired in series to the shutter and the experimenter's control panel. This ensured that the shutter opened only when the subject's head was correctly aligned in the head rest and when the experimenter's switch was on. The reduction screen was illuminated by 4x6v bulbs inside the chamber, thus making the contour of the aperture clearly visible.

The real surface (condition 1) consisted of an open grid pattern made up from horizontal and vertical lengths of elasticised black thread stretched on a wooden frame 126cm high and 84cm wide. The spacing between adjacent rows and columns was 8.5cm. This formed a regular grid pattern of 8.5cm x 8.5cm squares. The wooden frame was suspended between two upright 'Dexian' supports mounted on a 'Dexian' base. The wooden frame could be slanted backwards or forwards about its central horizontal axis and the angle of slant was measured on a 360° protractor fixed to the frame. This surface was viewed against a matt white painted wall which was located 200cm behind it. The wooden frame never appeared in the subject's field of view. Through the aperture, the real surface appeared as a grid made up of black lines against a white background.

For the picture surfaces (condition 2) the photographic slides were prepared by the system outlined in the Introduction to this chapter. These represented a surface made up of 8.5cm x 8.5cm squares located at a distance of 95cm from the eye, and at various angles of slant. These were back-projected onto a 120cm x 120cm square of permatrace (Admel) back-projection material mounted on a self-supporting frame. This screen was located 95cm from the observer's eye and the projected image was magnified to produce 8.5cm x 8.5cm squares for the 0° surface. A Kodak carouselSAV2000 projector was used, fitted with a zoom lens and driven by a Random Access projection system (Electrosonic Ltd.). The slides were projected through a red filter (Wratton No. 25a) fitted to the lens of the projector. This meant that the surface appeared as being made up of red lines against a black field. The filter produced more even illumination over the area of the image and darkened the black areas of the surface. No attempt was made to match the configuration of the real surface (black lines against white background) because back-projected slides made up of black lines on a white field would be too bright.

The red and black slides produced the best image (though not necessarily the most similar image) and it was this type of slide that was being tested against a real surface. A registration mark was also included on each slide and this was aligned with a red light-emitting diode attached to the bottom of the projection screen, but which was not visible to the subject. This registration ensured that a reasonable degree of vertical and horizontal alignment existed across all the different slides used.

The response device consisted of a 34cm x 28cm piece of hardboard painted matt black, with a white 2cm wide border. The board was pivoted about its central horizontal axis and was mounted in a vertical aluminium frame. This frame acted as the reference axis for the tilting board and it was fixed to a wooden base for support. When moved, the tilting board turned a 50ohm potentiometer fixed to the aluminium frame and wired as a Wheatstone resistance measuring system (see Appendix C). Settings of the response device could therefore be read by the experimenter on a Marconi digital voltmeter in terms of millivolts. When calibrated it was found that a lmV reading on the meter corresponded to .066 degrees of slant of the response device. The response device was located to the right of the subject, with the central pivot axis at the subject's eye level. The subject turned through 90° from the eye-piece shutter to set the response device each time a response was made.

The experimental room was divided in two by a partition with the subjects half fully illuminated. The experimenter's side of the room was darkened for the projected slide condition, but fully illuminated for the real surface condition.

Stimuli and Design

The slant angles tested were 50°, 30°, 10° backward slant (top slanted away from the subject), 0°, and 10°, 30°, 50° forward slant (top slanted towards the subject). A factorial design was used with two factors:

Factor A Real surface vs Picture

Factor B Backward slant vs Forward slant.

Repeated measures were used on both factors.

Procedure

The subject was seated at the head-rest eye-piece station and the head and chin-rest was adjusted until the subject reported that two cross-sights visible through the eye-piece, were aligned. These cross-sights consisted of elasticised thread stretched across the central axis of the aperture and across the back-projection screen. When alignment was completed the cross-sights were removed and the shutter closed. The subjects were told that they would see through a round window, a surface made up of horizontal and vertical lines. This surface they were told, would appear to be slanted either away from them or towards them or straight up and down. Their task was to set the plane of the response board so that its slant matched the slant of the test surface. They were permitted to look back and forth between the response device and the test surface until they were satisfied with their judgement. At that point, they were to press a button mounted near the response device, which signalled the experimenter to record the response and to prepare for the next trial. The subjects were randomly assigned to two groups, one beginning with condition 1, the other beginning with condition 2. Under both conditions, each subject began with three practice trials, using test angles of 25°, 35° and 45°, followed by random presentation of the seven test angles. This was repeated three times with a 3-5 minute rest period between replications, thus three judgements were made for each angle and the mean of these three was used in the analysis. After one condition was completed, the subject left the room while the apparatus was changed for the new condition. Subjects were simply told that the surface would appear different but their task was the same.

## 5.3 Results

### Hypothesis 1

A separate analysis of variance was carried out for each of the 10°, 30° and 50° test angles. The F ratios for factor A (real vs picture) were 1.85, 0.29 and 0.04 respectively, all non-significant ($F_{.95}(1,9) = 5.12$) (see Appendix D for summary tables). The data supports hypothesis 1 and no difference was found between slant estimates from the real surfaces and picture surfaces.

### Hypothesis 2

The means from each subject have been plotted against the actual slant angle used. Depicted on the same graph is the curve obtained from the model (predicted slant versus actual slant). (See figures 12a, b,c,d and table 2). The correspondence between the total mean for the 10 subjects and the predicted value is good for all angles, especially for the backward slant data, although large individual differences are apparent for some of the larger slant angles. Hypothesis 2 is supported by the data and the predicted values for the slant estimates are close to the actual estimates.

For factor B (Backward slant versus forward slant), the F ratios were 1.75, 3.2 and 3.6 for 10°, 30° and 50° respectively. These were all non-significant (see Appendix D for summary tables).

Separate t-tests were carried out on the 0° slant means with $H_o$: $\overline{x} = 0°$. t (real) = 0.401 (9 df) non-significant and t(picture) = 1.21 (9 df) also non-significant.

TABLE 2

| | | DATA | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | REAL | | | | PICTURE | | | |
| | | Backward | | Forward | | Backward | | Forward | |
| Actual Slant | Predicted Slant Estimates | $\bar{x}$ | sd | $\bar{x}$ | sd | $\bar{x}$ | sd | $\bar{x}$ | sd |
| 50° | 26.6 | 26.8 | 6.8 | 23.2 | 8.3 | 27.8 | 4.7 | 21.7 | 9.3 |
| 30° | 13.7 | 14.1 | 5.6 | 12.5 | 6.2 | 14.3 | 4.8 | 11.6 | 5.2 |
| 10° | 4.2 | 4.7 | 2.9 | 4.1 | 3.0 | 4.4 | 1.9 | 3.5 | 2.3 |
| 0° | 0 | -0.1 | 1.0 | -0.1 | 0.7 | -0.3 | 1.0 | -0.3 | 0.7 |

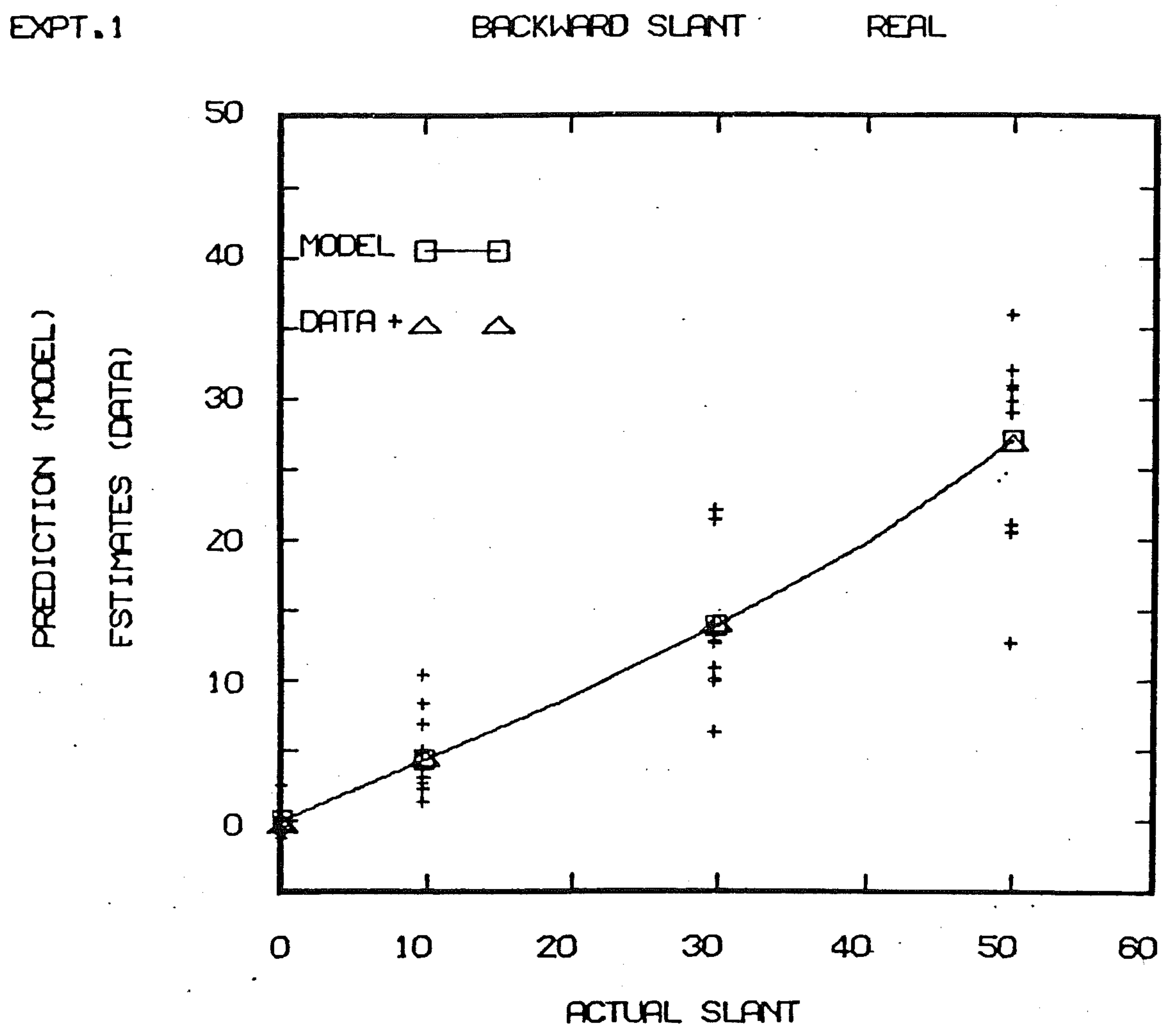



Figure 12. (a) Plot of actual slant against slant estimates. Included is the curve for the predicted values of beta.

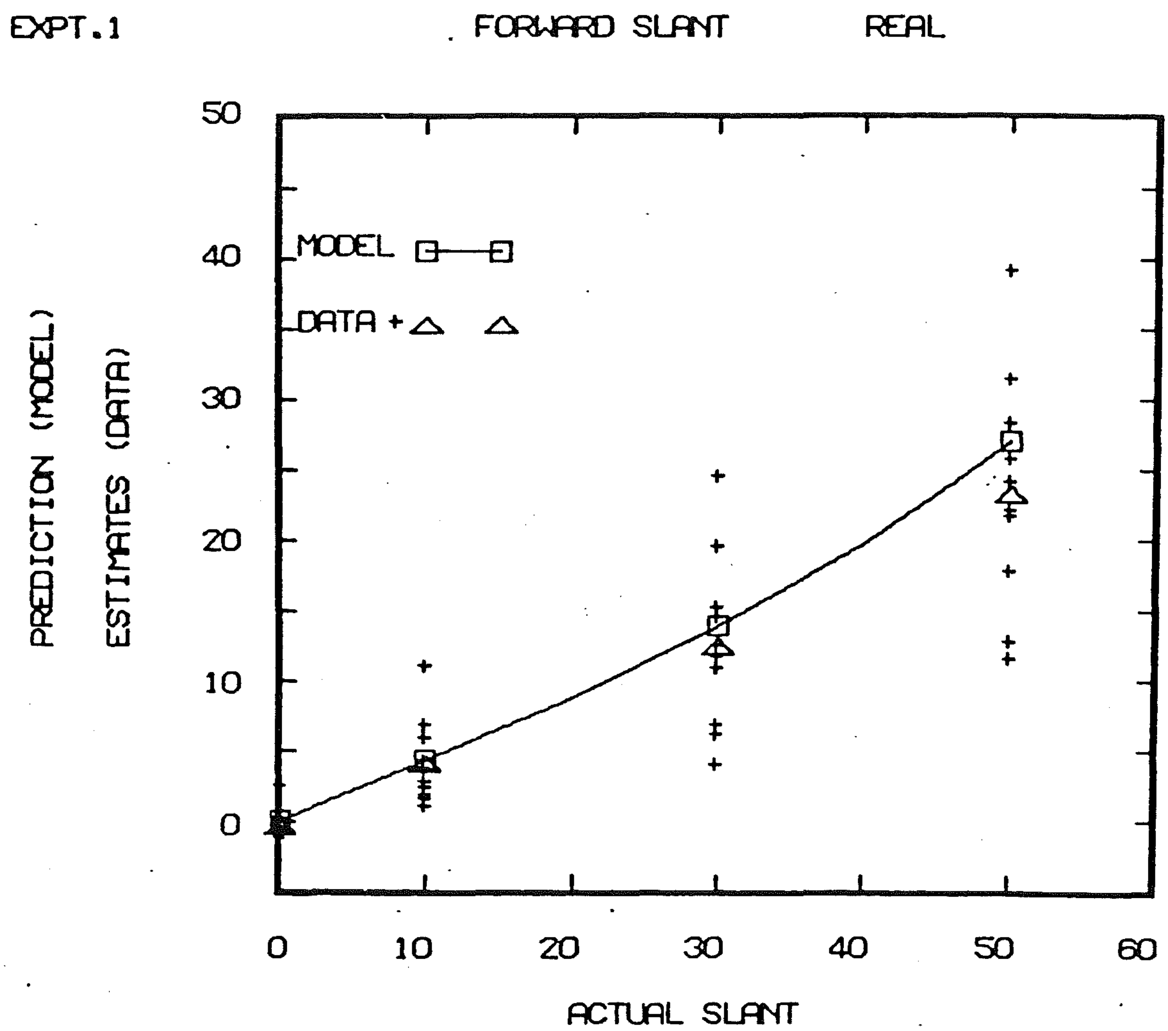

EXPT.1
FORWARD SLANT
REAL
MODEL
DATA +
PREDICTION (MODEL)
ESTIMATES (DATA)
ACTUAL SLANT
0
10
20
30
40
50
60

Figure 12 (b)

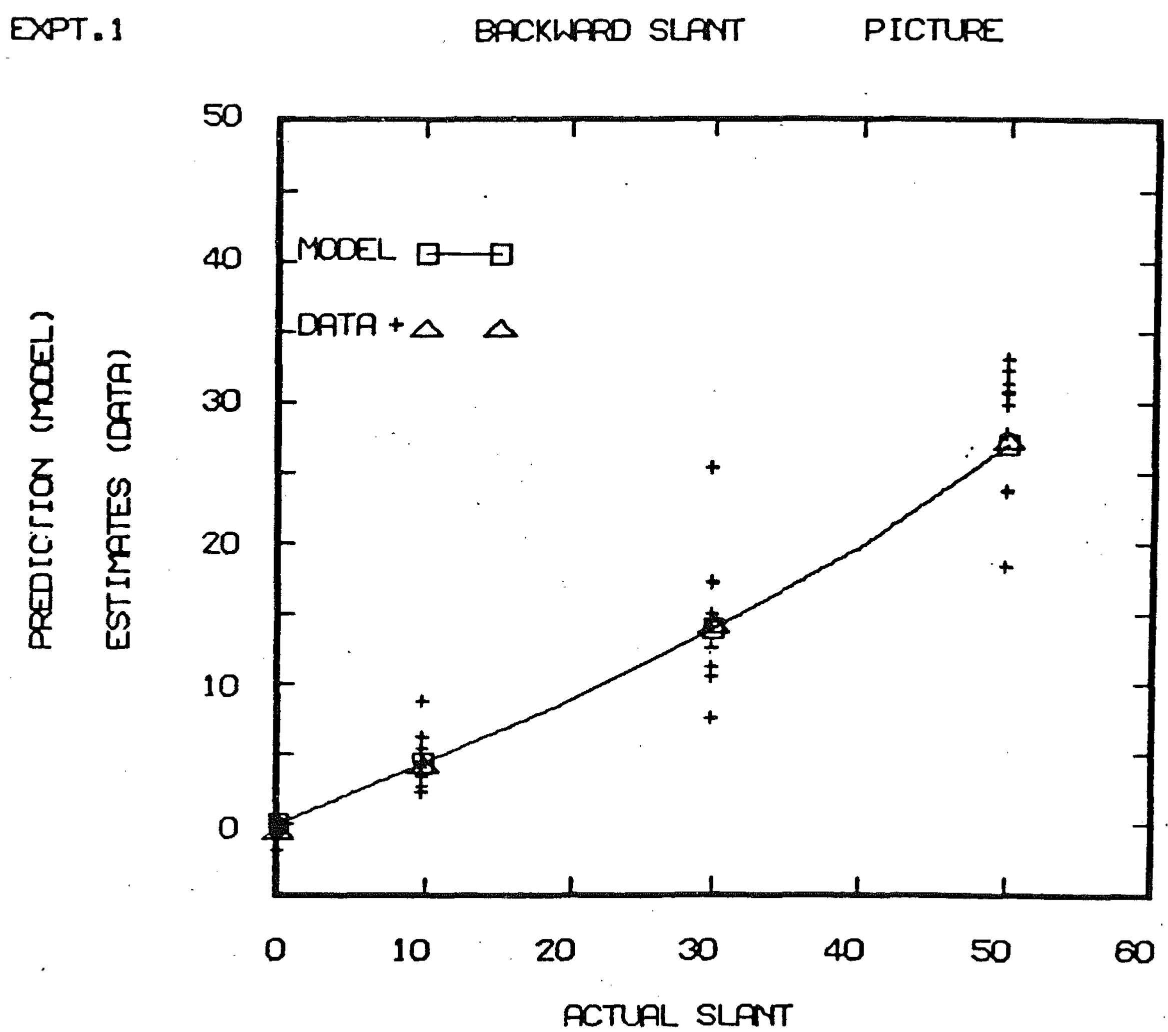
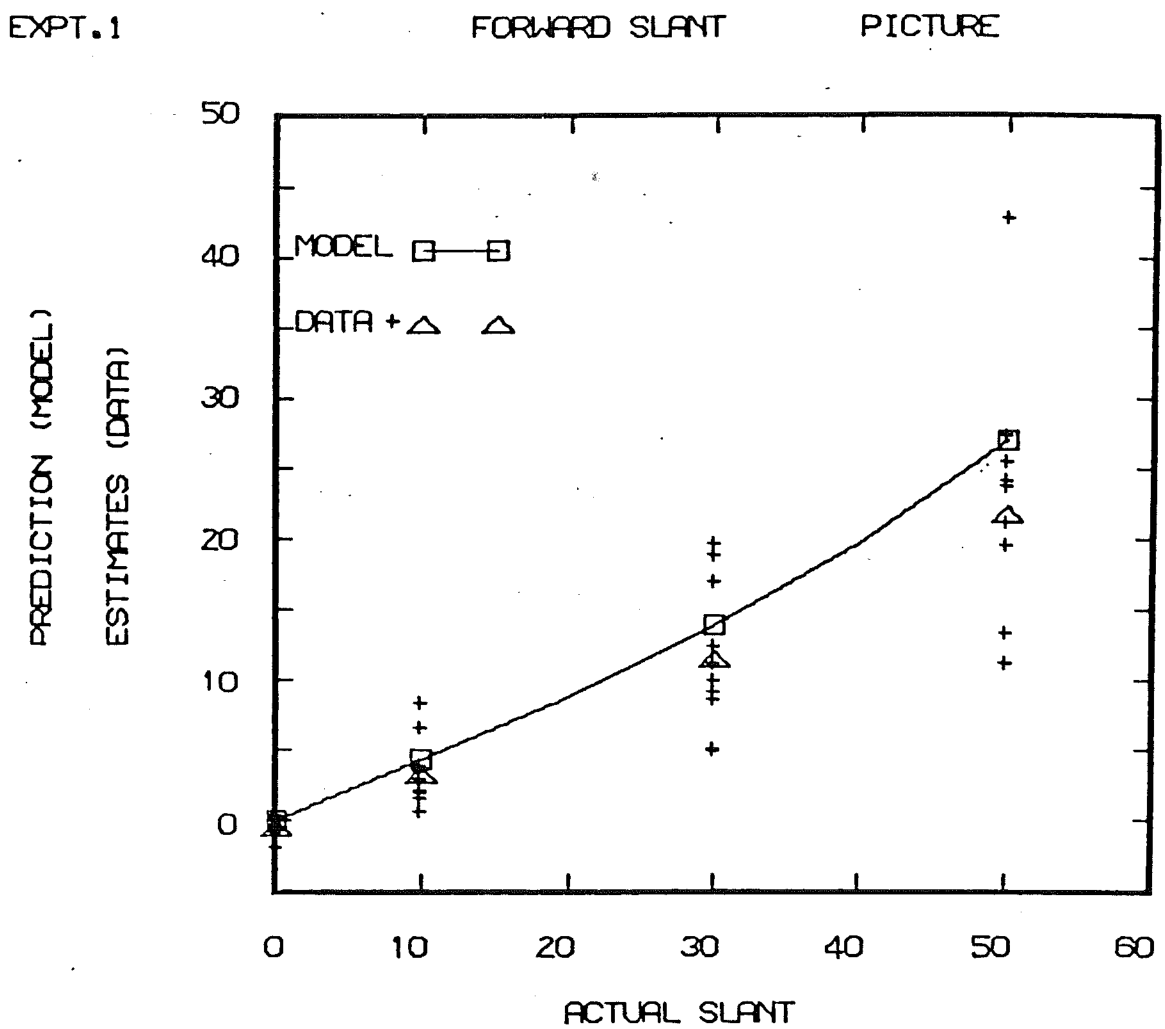

EXPT.1
BACKWARD SLANT
PICTURE
PREDICTION (MODEL)
ESTIMATES (DATA)
MODEL
DATA +
50
40
30
20
10
0
0
10
20
30
40
50
60
ACTUAL SLANT

Figure 12 (c)



EXPT.1
FORWARD SLANT
PICTURE
PREDICTION (MODEL)
ESTIMATES (DATA)
MODEL
DATA +
ACTUAL SLANT
0
10
20
30
40
50
60


Figure 12 (d)

## 5.4 Conclusions

No significant differences were found between the data from the real and picture test surfaces. Projected slide test surfaces will therefore be used in remaining experiments. This result also shows that explanations of underestimation based on the two-dimensional nature of the stimulus (e.g. Gibson 1950a) are not justified.

Comparison of the data with the predictions of the model shows that the model is an efficient predictor. Performance in the forward slant case seems to be even worse than that predicted by the model with the means lying below the predicted values. However, the difference between the backward slant and forward slant cases were found to be non-significant.

Given that the vertical acts as a reference point for the response device settings, we would expect greater variability of settings as the slant angle gets further from the reference point. It is easier to set the tilting board at 10° in relation to the vertical uprights than at 50° from the uprights. This could account for the greater variability of judgements that occurred at the larger slant angles.

Individual subject reports suggest that the majority of subjects were unaware that they were viewing a projected slide in condition 2, and most subjects expressed surprise when shown the apparatus behind the room partition. Subjects also expressed no great difficulty at carrying out the experimental task. The majority were confident of their judgements and this was reflected in the reasonably consistent results across the three replications.

The results support the predictions of the model for the case of aperture presentations. Typically for this type of experiment, the slant estimates have been expected by past researchers to match the actual slant angles of the test surfaces. Typically they are a lot less and underestimation occurs. It was hypothesised at the beginning of this experiment that subjects would generally evaluate the slant angle on the basis of the total visible length of the surface and the angle of convergence of the outermost visible perspective lines, according to the mechanism proposed in the model outlined in Chapter III. On this basis the slant of the test surfaces should be perceived to be less than the true slant and the actual values for the predicted perceived slant were calculated. The experimental data supported these predictions lending support to the hypothesis that the variables outlined in the model are the variables being used by subjects in making their judgements.

# CHAPTER VI

## EXPERIMENT 2

### 6.1 Introduction

The fact that the perceived straight-ahead deviates from the true straight-ahead is an essential part of the main underestimation model. Two possible means exist by which this effect could be tested. The first would be to test for distortion of the viewing aperture shape which is a predicted consequence of the deviation. However, my own personal observations indicate that although the distortion is detectable, it is not a strong effect and disappears when attention is directed at the aperture outlines rather than the surface. All in all it is a difficult phenomenon to test effectively. The second test is based on the fact that if deviation of the perceived straight-ahead occurs while a zero slant surface is being viewed, the surface will appear to be slanted slightly depending on the extent and direction of the deviation. Consider for instance, a situation in which an observer is presented with a surface at a large angle of slant, followed by a test surface at zero slant. It is possible that the deflected perceived straight-ahead direction from the first case, carries over to the zero slant surface and results in this surface being perceived as being slanted in the opposite direction to the slant of the first surface.*

*There is a similarity between this predicted result and the results of experiments testing the negative slant after-effect, e.g. Kohler and Emery, 1947; Bergman and Gibson,1959; Farne, 1970; Wenderoth, 1970. This is discussed in more detail in the final chapter.

It can be shown geometrically that deviation of the line of regard from the true straight-ahead direction results in a perspective gradient in the pattern of light reaching the eye. If it can be shown that an observer perceives a surface to be slanted, even though its physical slant is zero, we can presume that this is a result of the perceived straight-ahead direction deviating from the true straight-ahead direction. If we can show that this occurs, we have gained evidence for the primary mechanism of the underestimation model presented in Chapter III.

If an observer is presented with a series of backward slanted surfaces the model assumes that his perceived straight-ahead direction is downwards in relation to the true direction. If we then present this observer with a zero-slanted surface, then providing that the time intervals between presentations are not too great, one would expect that this downward direction of the straight-ahead would persist long enough to result in the zero-slant surface appearing to slant forward. For a series of forward slanted surfaces, we would expect the perceived straight-ahead to be above the true direction and the zero surface should appear slanted slightly backwards. For a backward slant series $\nu$ is negative in the equation $\phi = \theta - \nu$, and since $\theta = 0°$ then the predicted slant $\phi$, is positive. For a forward slant series, $\nu$ is positive, therefore $\phi$ is negative.

In experiment 1, the backward and forward slant test surfaces were randomised in their presentation, and so presumably any carry-over effect would have been cancelled out. It was found that the means for the zero slant judgements did not differ significantly from zero. Experiment 2 was designed such that the backward slant and forward slant test surfaces were presented separately. Embedded in the series of backward and forward presentations was a test slant of zero degrees. The hypothesis was that the mean judgement for the backward slant series, of the zero test surface would be a forward slant value greater than zero, and the mean judgement for the forward slant series of the zero test surface would be a backward slant value greater than zero. The experiment also enabled more data to be gathered for testing the predicted slant estimates, $\beta$, of the model.

## 6.2 Method

### Subjects

The subjects were another group of 10 students from the University of Canterbury, all volunteers. Six males and four females were used.

### Apparatus

The apparatus was identical to that used for the picture presentation (condition 2) section of experiment 1.

### Stimuli

The test surface was a 0° slant surface identical to those used in experiment 1.

Condition 1 was a series of backward slants: 10°, 20°, 30°, 40° and 50°, plus the 0° test surface.

Condition 2 was a series of forward slants, 10°, 20°, 30°, 40° and 50° plus the 0° test surface.

Under each condition the order of presentation of the angles was randomised with the zero test surface appearing at any point within the series.

### Procedure

The general procedure was similar to that used under condition 2 of experiment 1. Subjects were randomly assigned to two groups, one beginning with condition 1, the other beginning with condition 2. Subjects were run over two sessions, with approximately one week between each of the two sessions; one condition was run per session. Each subject began a session with three practice trials using separate test angles (25°, 35°, and 45°), followed by random presentation of the test series. This was repeated three times with a 3-5 minute rest period between replications. No time limits were set on the time the subject could view a test surface (inspection time), nor the time taken to make the judgement (time between trials), although the importance of these factors upon the slant after-effect was realised. The aim of the experiment was to test for deviation of the apparent straight-ahead in the context of a normal slant perception experiment.

## 6.3 Results

For condition 1 (backward slant series), the mean perceived slant for the 0° test surface was 1.81° (sd=1.55) in a forward position. This was found to be significantly greater than zero. $t = 3.7 > t_{crit}$ (.005, 9df) = 3.25, and so the data supports the hypothesis.

For condition 2 (forward slant series) the mean perceived slant for the 0° test surface was 1.18° (sd=2.87) in a backward direction. This was in the hypothesised direction, but was not significantly greater than zero $t = 1.299 < t_{crit}$ (.1, 9df) = 1.383.

The estimated slant data is plotted against the actual slant angle in figures 13a, b. The curve for the predicted slant angle $\beta$, as a function of actual slant is also shown. Once again, the model is a good description of the data. The means and variance from the data are shown in Table 3.

An analysis of variance (1 factor, repeated measures) was carried out for each angle testing for a difference in slant estimates from the backward slant and forward slant conditions.

| | | | | |
|---|---|---|---|---|
| For: | 50° | F = 4.56 | n.s. | F (1,9) = 5.12 |
| | 40° | F = 9.4 | significant | |
| | 30° | F = 5.06 | n.s. | |
| | 20° | F = 11.44 | significant | |
| | 10° | F = .04 | n.s. | |

Judgements for backward slant test surfaces were generally greater than those for forward slant test surfaces, but only for 40° and 20° were these differences significant (see Appendix D for ANOVA summary tables).

TABLE 3

| Actual Slant | Predicted Slant | DATA | | | |
|---|---|---|---|---|---|
| | | Backwards | | Forwards | |
| | | $\bar{x}$ | sd | $\bar{x}$ | sd |
| 50° | 26.6 | 29.7 | 7.2 | 24.0 | 6.5 |
| 40° | 19.5 | 24.0 | 6.7 | 17.8 | 5.8 |
| 30° | 13.7 | 14.6 | 5.5 | 10.7 | 3.7 |
| 20° | 8.7 | 7.8 | 1.8 | 5.4 | 3.1 |
| 10° | 4.2 | 1.9 | 1.6 | 1.7 | 2.3 |

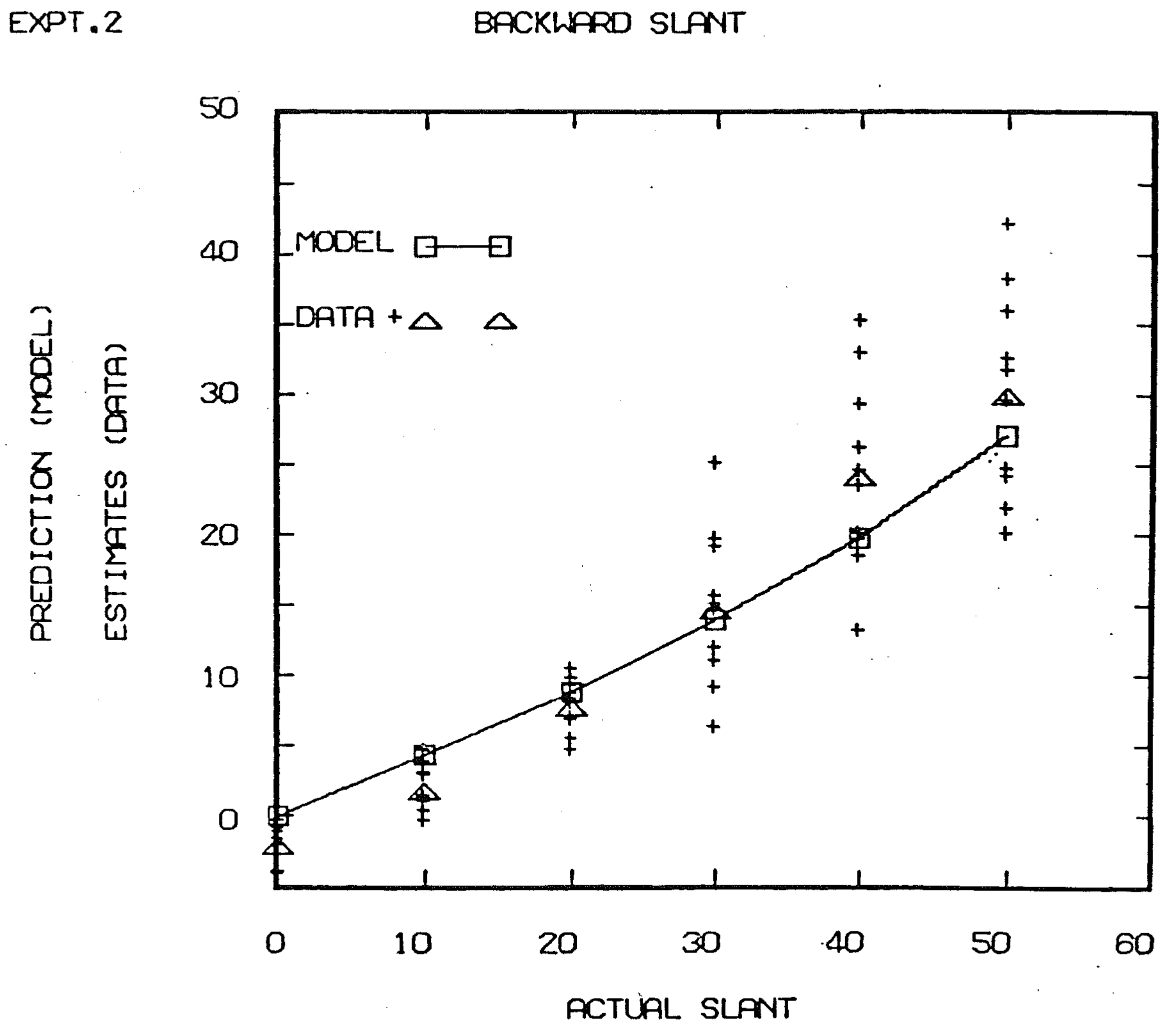



Figure 13 (a) Plot of actual slant against slant estimates. Included is the curve for the predicted values of beta.

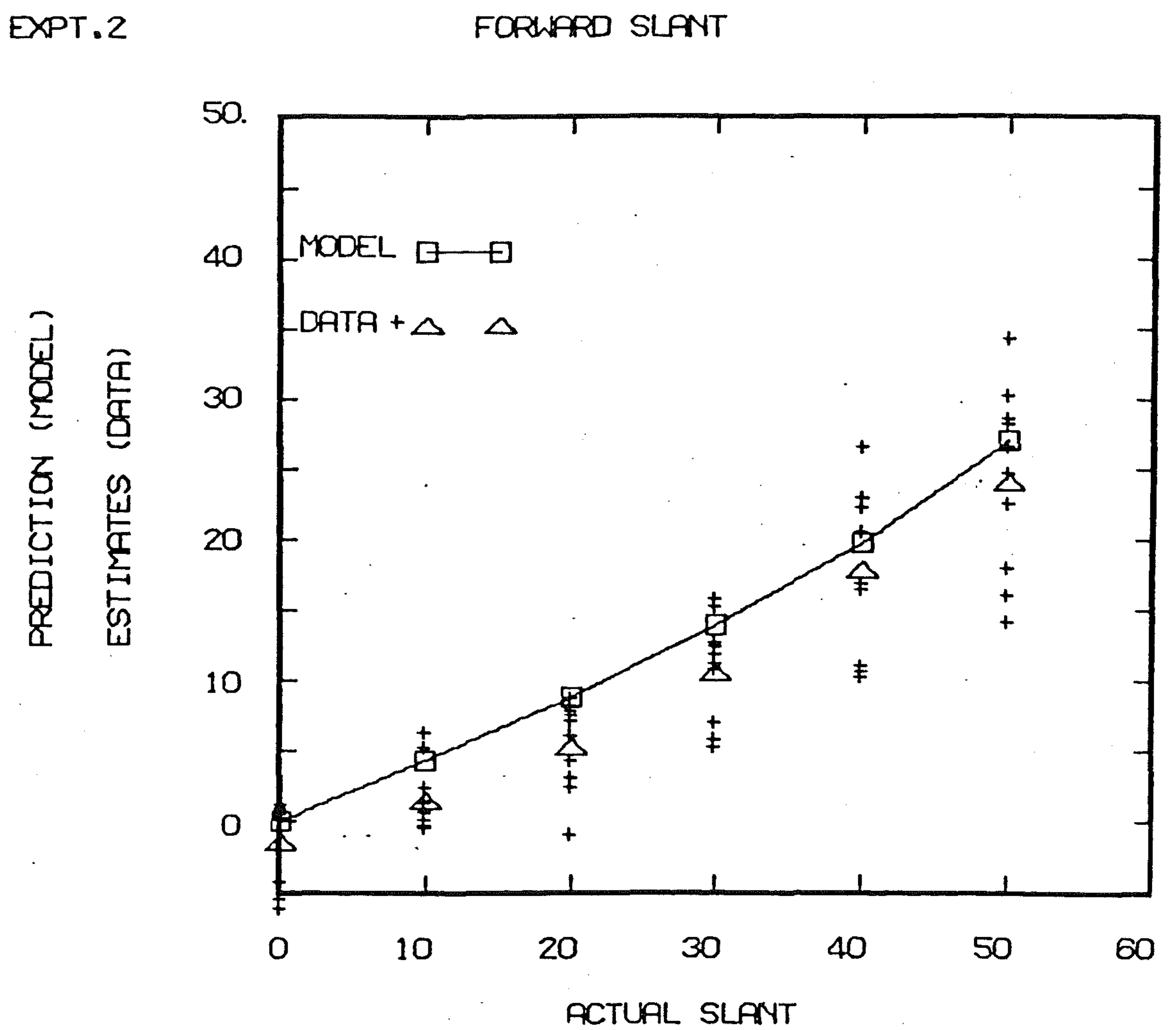

EXPT.2
FORWARD SLANT
MODEL
DATA +
PREDICTION (MODEL)
ESTIMATES (DATA)
ACTUAL SLANT
50.
40
30
20
10
0
0
10
20
30
40
50
60

**Figure 13 (b)**

## 6.4 Conclusions

Partial evidence was gained for the hypothesis that the perceived straight-ahead direction deviates from the true position and results in slant being perceived when a zero-slant surface is viewed. The forward slant case was in the expected direction, but was not significant. The difference in results from the backward slant condition and the forward slant condition may be a reflection of the basic differences that exist in our environment between the distribution of information in our visual fields, above and below our eye-level. It may also be a result of some basic physiological difference between an upward gazeland and a downward gazeline. Surfaces slanted forward from the vertical are rare in our everyday perceptual world (because of gravity) whereas backward slanted and vertical surfaces are quite common. This may produce a reluctance on the part of the observer to perceive a forward slanted surface as such, and may account for the different results from the two conditions.

To test for these factors properly, one would need to repeat the experiment using surfaces slanted about a vertical axis. Kleinhans (1970) obtained differences in results between judgements of perceived ego-centric location made in the up-down direction and those made in relation to the medial plane (left-right direction). It is not surprising therefore that different results have been obtained for slant judgements made in the backward direction and those for the forward slant direction.

The general trend of the data is close enough to the predictions of the model to once again lend support to the underestimation model. As in experiment 1, the means for the forward slant condition tend to be below the predicted values. This may be attributable to differences between 'up' and 'down' that exist in our environment as discussed above or it may be a tendency for some subjects to use the sub-optimum strategy of not using the full width of the surface visible at the diameter of the aperture, in their evaluation of $\theta$, but rather the smaller extents visible at the top or bottom. This would account for the correct trend in the data, but a constant decrement between the predicted value and the mean of the judgements. The large individual differences in the results, however, prevent any definite conclusions being reached regarding any systematic differences between the backward and forward situations.

Some evidence was gained for the proposal that perceived straight-ahead deviates from the true straight-ahead. This proposal was tested under the most 'difficult' conditions with no control of time between presentations, nor the number of pre-test surfaces presented before the zero-slant surface. Had all subjects been required to view the same number of slanted surfaces before the test surface, and after a limited time interval, the results may have been more conclusive. Even if no evidence had been gained for the proposed deviation of the straight-ahead direction the closeness of the predictions of the model and the data indicate that the variables outlined in the model are those being used by the subjects. The question would remain as to why these variables are used, instead of the 'correct' ones. I believe that sufficient evidence exists to retain the original proposal related to deviation of the straight-ahead direction.

# CHAPTER VII

# EXPERIMENT 3

## 7.1 Introduction

Experiments 1 and 2 provided good support for the underestimation model presented in Chapter III. These experiments used test surfaces which consisted of large squares with outlines clearly delineated, and the surfaces were viewed through an aperture located in such a position as to appear in focus and well defined. It could be argued that these are special conditions and are unlike those used in many slant perception experiments where the aperture is closer to the eye and complex texture patterns are used.

In order to meet this argument and to extend the generality of the model an experiment was designed which used test surfaces made up of a large number of small units and the aperture was moved to a position closer to the eye. The size of the aperture was correspondingly decreased to maintain the same field of view used in experiments 1 and 2.

Test surfaces were generated which were equivalent to a surface made up of squares measuring 3.2cm x 3.2cm located at a distance of 95cm from the observer. The predictions of the model generated by program 2 (Appendix A) for these new conditions are:

| | | | | | | |
|---|---|---|---|---|---|---|
| $\theta$ | 0° | 10° | 20° | 30° | 40° | 50° |
| $\beta$ | 0° | 4.79° | 9.82° | 15.35° | 21.75° | 29.53° |

The model was tested over 0°, 10°, 20°, 30° and 50° for both forward and backwards conditions. The hypothesis states that the slant estimates will lie close to the values of $\beta$ given above.

## 7.2 Method

### Subjects

Subjects were eight psychology Stage 1 students at the University of Canterbury, all volunteers (three males and five females).

### Stimuli

Nine angles were tested, 0°, 10°, 20°, 30° and 50° backwards and forwards. These were presented in random order with backward and forward slants mixed.

### Apparatus

This was the same as that used in experiments 1 and 2, except that the aperture was moved to a position 14cm from the subjects eye. It consisted of a 6cm diameter circular hole cut in a sheet of white card-board. This gave a 24° field of view, as in experiments 1 and 2. However unlike experiments 1 and 2 the reduction screen in this experiment was not illuminated.

### Procedure

Three practice trials preceeded the presentation of the test angles and three judgements were made for each angle.

The basic procedure followed that for experiments 1 and 2.

## 7.3 Results

The means and variances are presented in table 4.

These means are plotted as a function of actual slant angle in figures 14a, b, along with the predictions of the model for this particular experimental set-up. The model still provides a good predictor for the data, even though the new experimental conditions were used and so once again the hypothesis is supported. As in experiments 1 and 2, the mean judgements are slightly below the predicted values for the forward slant condition. An analysis of variance was carried out to test for differences between forward and backward slant judgements for each of the angles, 10°, 20° 30° and 50°.

The F values were:

| | | | |
|---|---|---|---|
| 10° | F = .05 | n.s. | $F_{95}$ (1, 7) = 5.59 |
| 20° | F = .22 | n.s. | |
| 30° | F = 2.61 | n.s. | |
| 50° | F = 15.74 | significant | |

See Appendix D for ANOVA summary tables.

For the 50° test angle, the judgements were significantly greater for the backward slant condition than for the forward slant condition.

The mean judgement for the 0° test surface was -.19 (sd = 3.7). A t-test yielded a value of $t = .147 < t_{crit}$ (.05, 7df) = 1.895, indicating a non-significant difference from zero.

TABLE 4

| Actual Slant | Predicted Slant | DATA | | | |
|---|---|---|---|---|---|
| | | Backwards | | Forwards | |
| | | $\bar{x}$ | sd | $\bar{x}$ | sd |
| 50° | 29.5 | 33.8 | 6.7 | 25.1 | 6.2 |
| 30° | 15.3 | 16.6 | 6.2 | 12.3 | 5.5 |
| 20° | 9.8 | 9.5 | 6.7 | 7.6 | 6.3 |
| 10° | 4.8 | 2.9 | 5.7 | 2.2 | 3.7 |
| 0° | 0.0 | -0.2 | 3.7 | -0.2 | 3.7 |

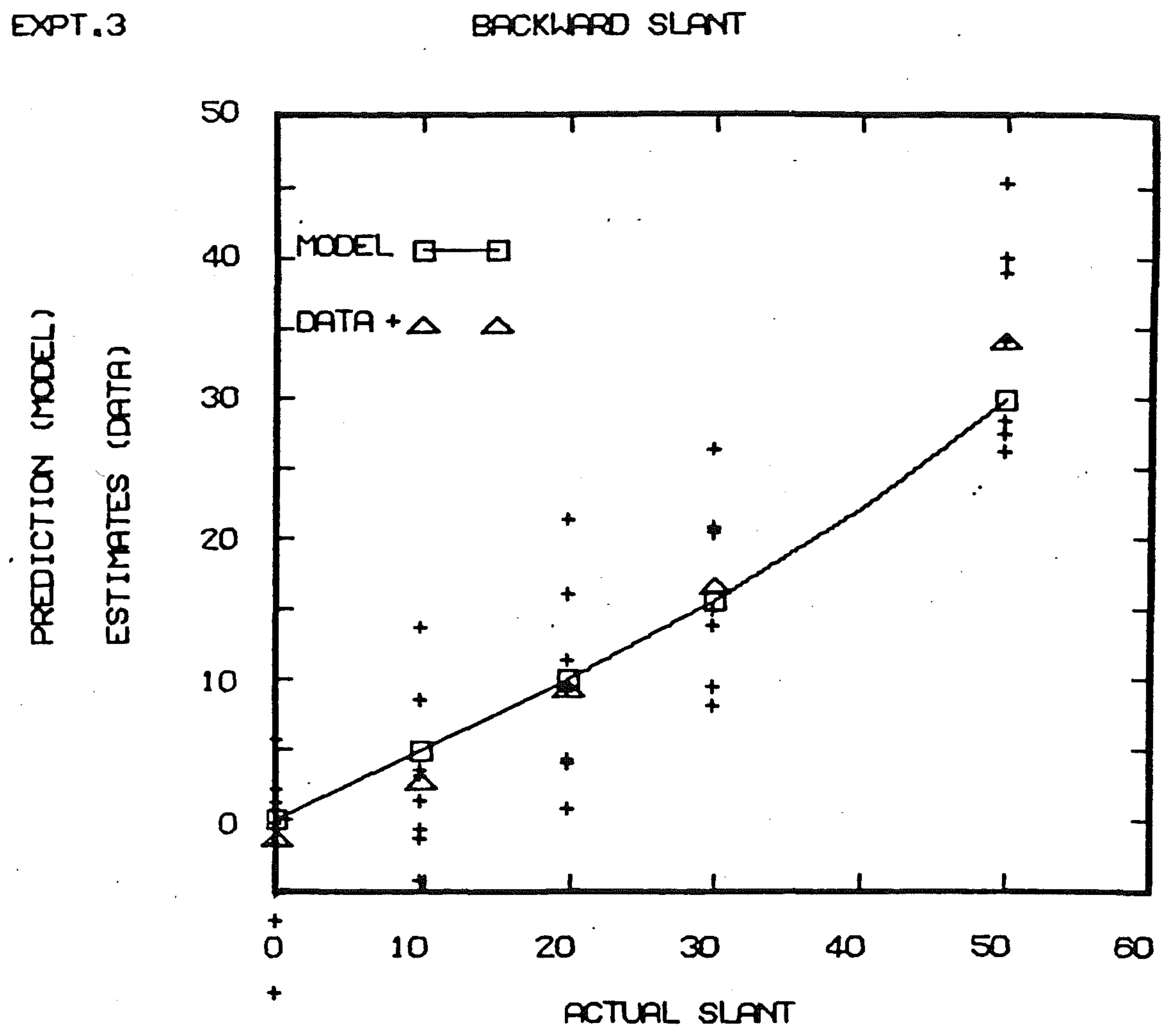



Figure 14 (a) Plot of actual slant against slant estimates. Included is the curve for the predicted values of beta.

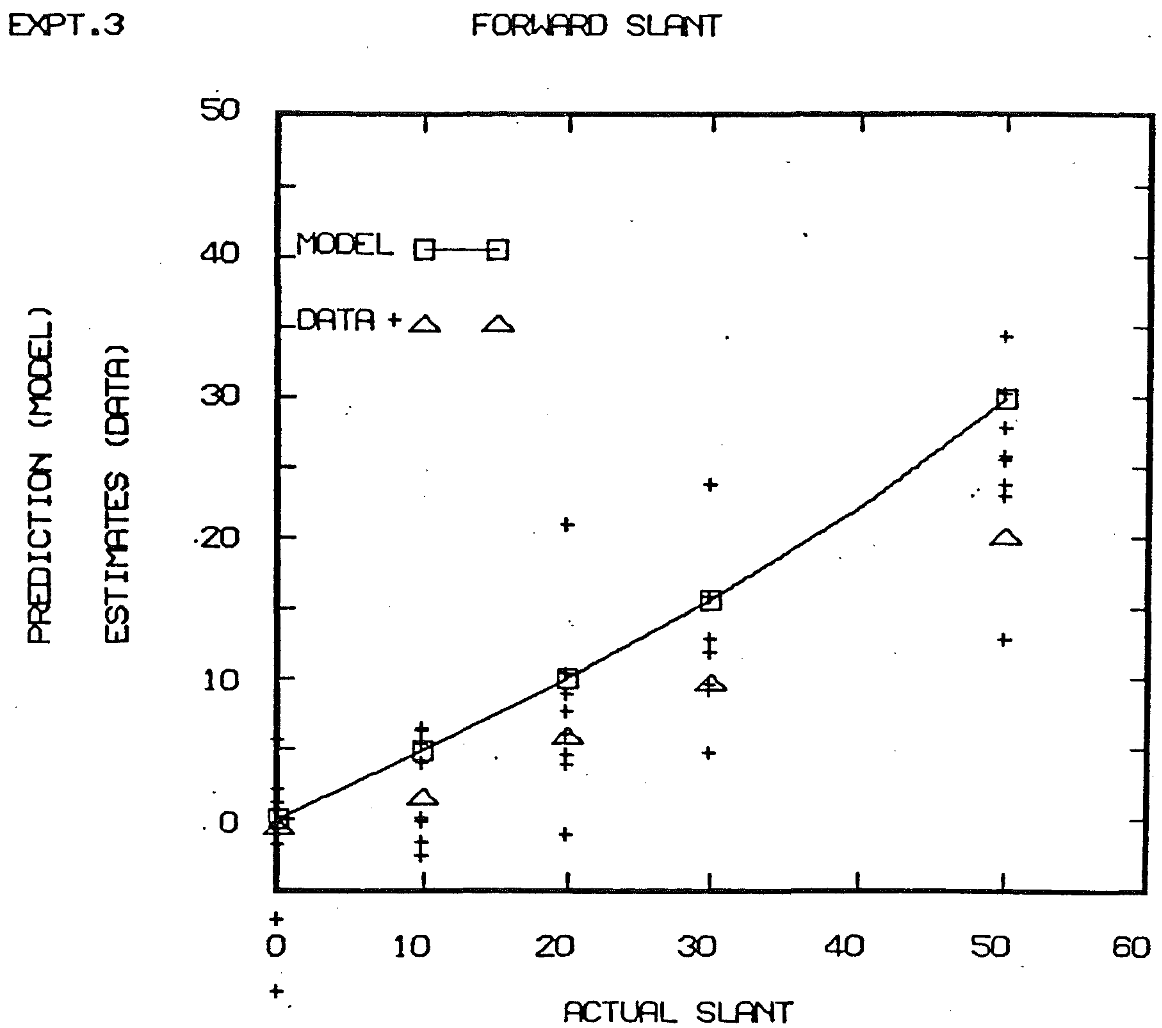

EXPT.3
FORWARD SLANT
MODEL
DATA +
PREDICTION (MODEL)
ESTIMATES (DATA)
ACTUAL SLANT
0
10
20
30
40
50
60

Figure 14 (b)

## 7.4 Conclusions

Even though the experimental conditions were changed to a large degree, the results were still in keeping with the predictions of the model. The test surfaces were such that simple 'outline' figures were not obvious on the surface and the model seems equally valid under these 'more difficult' conditions. Comparison of figures 14a and b with figures 13a and 13b in experiment 2 shows a greater degree of scatter of the data points in the former case. This can be explained by considering that with the aperture closer to the eye, it only takes a very small displacement of the eye from the observation point to produce quite large differences in the visible areas of the surface and consequently the slant judgements. Although care was taken to ensure that subject's heads were correctly positioned in the head-rest, there was still opportunity for some change in the position of the head and exact alignment for all subjects was impossible without resorting to some type of head-clamp system.

Since the backward and forward slant angles were mixed, we do not expect the 0° test slant judgements to differ from zero and this was found to be the case.

# CHAPTER VIII

## Experiment 4

### 8.1 Introduction

The previous experiments do not preclude the possibility that an alternative explanation could account for the data. A more rigorous test of the model would result if changes to the parameters of the model produce measurable changes in the data.

One of the relevant parameters in the case of aperture presentation, is the size of the aperture. However, as pointed out in Chapter III, the maximum predicted perceived slant can never exceed $\tan^{-1} (\tan\theta)/_2$ for a surface slanted at $\theta$ degrees through a circular aperture.

For a given size of texture unit, the predicted slant reaches a maximum and then falls off to a minimum value with changes to the size of the field of view. This goes in a cyclic pattern with the upper limit represented by the maximum given above, and a lower limit dependent upon the size of the texture elements. For example, the predicted values for perceived slant with a changing field of view are listed below. The stimulus conditions correspond to those used in experiment 3, i.e. D = 95cm, size of units = 3.2cm x 3.2cm. The angle of slant is 50°.

| Field of view (degrees) | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|
| $\beta$ predicted slant | 29.5 | 28.5 | 27.5 | 30.3 | 29.4 | 28.5 | 27.6 |

In this case the maximum difference in predicted perceived slant will never exeed 3°, no matter how much the field of view is changed.

Given the variance that was evident in the preceeding experiments, such a small difference would be undetectable using the experimental techniques adopted previously. It is for these reasons that simple changes to the field of view were not used as a test of the model.

Examination of the equation for rectangular apertures reveals that the predicted slant estimates are dependent upon both the width and height of the aperture. Changes to the shape of the aperture should affect the values of w and L used by the observer in his/her evaluation of the slant angle.

The general equation for the slant underestimation model is given by equation [A] i.e.

$$\beta = \tan^{-1}\left(\frac{w\sin\theta.(D^2-L^2\sin^2\theta)}{4LD^2\cos^2\theta}\right)$$

If we start off with a square aperture, we can use program 3 (Appendix A) to determine the values of w and L visible to the observer, and the predicted value for $\beta$. Now in equation [A] an increase in the value of w results in a larger value for $\beta$. If we increase the width of the aperture, we are effectively increasing the value of w, without changing any other variables in equation [A]. Therefore, by using a rectangular aperture which has the same height as the square aperture, but greater in width, we should obtain greater slant estimates than for a square aperture (Hypothesis 1).

If we use a rectangular aperture that is the same width as the square aperture, but greater in height, we increase the value of w (see equation [2], Chapter III), which means an increase in the value of $\beta$, but we also increase the value of L. An increase in L reduces the value of $\beta$ by an amount greater than the increase in $\beta$ caused by the increase in w.

(This is because of the term $L^2$ in equation [A]). The net result is a predicted value of $\beta$ which is less than that for the square aperture. In other words, by changing from a square aperture to a long narrow aperture which has the same width as the square aperture but is greater in height, we should obtain slant judgements slightly less than those for the square aperture and less than those for the wide rectangular aperture (Hypothesis 2).

The square aperture used had a 24° field of view in both the vertical and horizontal directions (condition 1).

The wide rectangular aperture had a 32° field of view in the horizontal direction and a 24° field of view in the vertical direction (condition 2). The long rectangular aperture had a 24° horizontal field of view and a 32° vertical field of view (condition 3).

'Horizontal' corresponds to the direction parallel to the axis of rotation of the test surfaces. The same test surfaces used in experiment 3 were used, i.e. D = 95cm, and the units measured 3.2cm x 3.2cm. Two angles were tested, $\theta = 50°$ and $\theta = 20°$. The stimulus conditions for $\theta = 50°$ are depicted in figure 15. Notice the differing amounts of surface extent visible in each aperture. It is the different values of w and L visible in each case that the model predicts will result in specific differences in slant judgements.

The predicted values of the slant estimates are obtained from program 3 (Appendix A). These are:

| | Condition 1 | Condition 2 | Condition 3 |
|---|---|---|---|
| 50° | 37.1° | 46.1° | 32.2° |
| 20° | 9.8° | 14.6° | 8.5° |

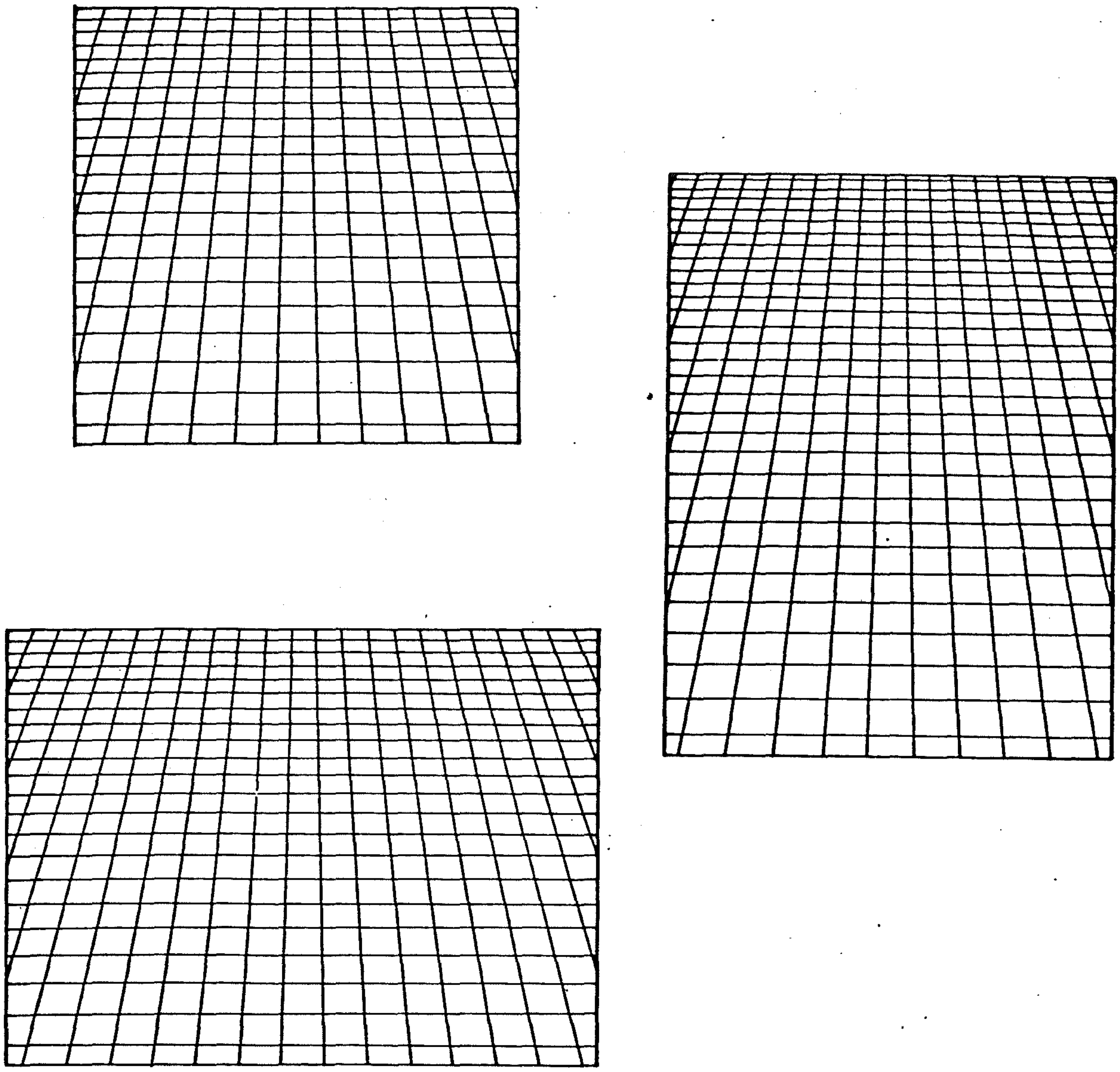


Figure 15. Two dimensional representation of the information available to the observer through the different types of apertures used in experiment 4. Distortion of the outlines of the figures is also evident, particularly if the page is turned through 90°. This distortion complies with that predicted in ch. 4 and has similarities to Ehrenfels (1890) variant of the Ponzo illusion.

Notice also that for the 50° slant, the predicted value for the slant estimate for a square aperture of 24° field of view (condition 1) is greater than that for the circular aperture of 24° field of view used in experiment 3 ($\beta = 29.5°$). This is hypothesis 3.

## 8.2 Method

### Subjects

Subjects were eight students from the University of Canterbury, three males and five females, all volunteers.

### Stimuli

Two test angles were used, 20° and 50° backward slant only. The test surfaces were identical to those used in experiment 3.

The two test angles were run under each of the three conditions.

### Apparatus

For condition 1, the aperture measured 6cm x 6cm and was located 14cm from the subject's eye. For condition 2, the aperture was a rectangle measuring 8cm wide x 6cm high, and for condition 3 it was a rectangle 6cm wide x 8cm high. The aperture was always centred around the subject's true straight-ahead direction.

The remaining apparatus was identical to that used in experiments 2 and 3.

### Procedure

The three conditions were counterbalanced for order and practice effects. The subject made two judgements of each of the two test angles under the three different conditions. Three practice trials preceeded each set of judgements and test surfaces representing, 10°, 30° and 40° (backwards) were randomly interspersed amongst the test angles. This was to prevent set formation with just the two test angles and the data from these extra angles was not recorded. A rest period of five minutes occurred between each condition and subjects were told that the window through which they were looking would be different in each case, but their task remained the same.

## 8.3 Results

The means from each of the subjects two judgements are shown in Table 5, along with the means and standard deviations for each of the conditions.

Wilcoxan matched pairs signed rank tests as outlined by Siegel (1959) were carried out between the various conditions to test hypotheses 1 and 2.

For hypothesis 1, we have

$H_o$ : no increase in mean judgements between conditions 1 and 2

$H_1$ : mean judgements for condition 2 greater than mean judgements for condition 2.

For 50° $T = 1$ ($N = 8$), therefore reject $H_o$ at .01 level of significance (1 tailed).

For 20° $T = 3$ ($N = 8$), reject $H_o$ at .05 level (1 tailed).

For hypothesis 2 we have:

$H_o$ : no decrease in judgements between conditions 2 and 3

$H_1$ : mean judgements for condition 3 less than mean judgements for condition 2.

For 50° $T = 1$ ($N = 8$), reject $H_o$ at .01 level (1 tailed).

For 20° $T = 4$ ($N = 8$), reject $H_o$ at .05 level (1 tailed).

Tests to see whether the mean judgements for condition 3 were less than those for condition 1 were non-significant for both 50° and 20°.

For hypothesis 3, we have from experiment 3, with 24° field of view, circular aperture $\overline{x}_c = 33.83°$ sd = 6.99 for the 50° backward slant test surface.

For experiment 4, with 24° square aperture, $\overline{x}_s = 40.38°$, sd = 5.82

$H_o : \overline{x}_s = \overline{x}_c$

$H_1 : \overline{x}_s > \overline{x}_c$

A t-test (independent samples) yields a t value

$$= 2.038 > t_{crit}\ (.05,\ 14df) = 1.761$$

Therefore reject $H_o$ at .05 level.

The means from the eight subjects are plotted in figure 16 against the three different conditions. A predicted inverted u-shaped function is also shown and the data can be seen to have this same property.

TABLE 5

| | Subject | | Cond. 1 Degrees | Cond. 2 Degrees | Cond. 3 Degrees |
|---|---|---|---|---|---|
| | 1 | | 29.5 | 33.0 | 29.3 |
| | 2 | | 42.4 | 44.5 | 32.7 |
| | 3 | | 44.5 | 59.9 | 37.6 |
| 50° | 4 | | 43.5 | 44.9 | 41.8 |
| | 5 | | 34.9 | 35.4 | 36.2 |
| | 6 | | 47.3 | 55.4 | 50.7 |
| | 7 | | 38.3 | 54.8 | 54.5 |
| | 8 | | 42.7 | 42.6 | 38.5 |
| | | $\overline{x}$ | 40.4 | 46.4 | 40.2 |
| | | sd | 5.8 | 9.7 | 8.6 |
| | 1 | | 11.8 | 14.3 | 12.1 |
| | 2 | | 13.2 | 20.6 | 13.6 |
| | 3 | | 2.3 | 36.4 | 6.0 |
| 20° | 4 | | 10.6 | 13.1 | 8.2 |
| | 5 | | 14.7 | 20.3 | 22.7 |
| | 6 | | 19.9 | 23.4 | 10.6 |
| | 7 | | 11.6 | 17.6 | 11.7 |
| | 8 | | 11.9 | 8.8 | 10.4 |
| | | $\overline{x}$ | 12.0 | 19.3 | 11.9 |
| | | sd | 4.9 | 8.4 | 5.0 |

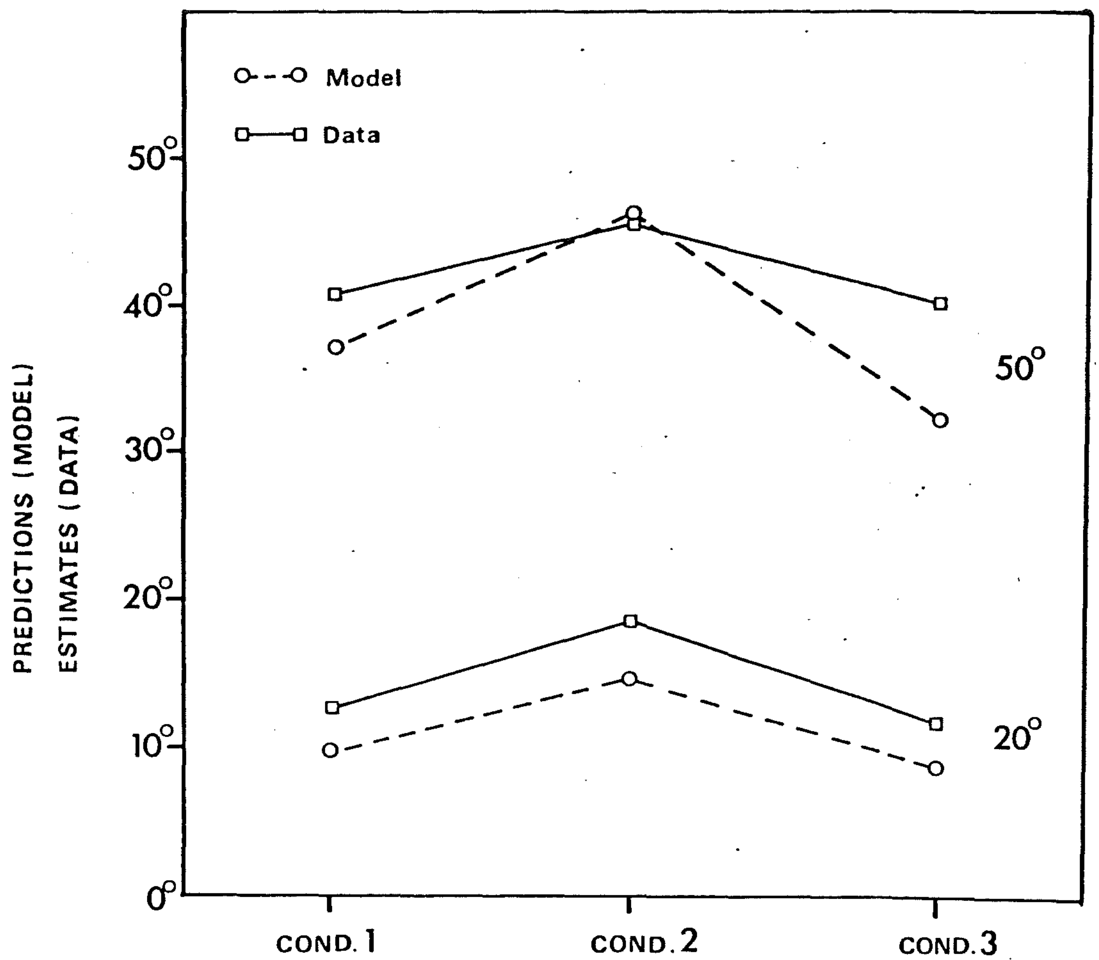



Figure 16. Plot of the mean slant estimates for the three different experimental conditions. The predicted values from the model are also included.

## 8.4 Conclusion

All the hypothesised changes predicted by the model were substantiated. Increasing the width of the aperture (condition 2) without changing the height increases the amount of surface visible to subjects in the horizontal direction and so the value of w increases. The model predicts that a corresponding increase in the slant judgements should occur. The results supported this prediction. Increasing the height of the aperture without changing the width should produce slightly smaller slant judgements, but judgements quite a bit smaller than those for the wide aperture. The results supported the latter prediction but not the former. However, the size of the predicted difference was only 5° for the 50° slant and only 0.3° for the 20° slant. Such small differences would be difficult to detect using the techniques adopted here since the smallest variance from this experiemnt was about 23° (condition 1, 20°, sd = 4.9). The model also predicts a difference in the slant estimates for a circular aperture and a square aperture of the same field of view since the difference in shape between these two types of apertures introduce specific changes to the parameters in the model. The results supported this prediction.

When the results from experiments 1, 2, 3 and 4 involving circular and rectangular apertures are combined with the existing empirical data from experiments involving isolated figures (Clark et al, 1955; Smith, 1956, 1959, 1966), we find that the model is a good all round predictor and very general in its application. Large individual differences do exist in the data and not every subject uses the same strategy in making his judgement. However, on average the model enables us to predict with a reasonably high degree of accuracy the amount of underestimation that will occur in a given situation. It also points to the factors that we need to control in order to control the amount of underestimation. This means presumably, that if we select the correct combination of aperture and surface we could even produce slant overestimations.

# CHAPTER IX

# SUMMARY AND CONCLUSIONS

## 9.1 The Experiments

In experiment 1 both a real surface and a two-dimensional representation of a surface were used to test the predictions of the model. The subjects viewed the surfaces, at various angles of slant, through a circular aperture with a 24° field of view. For both the real surfaces and the picture surfaces the judged slant was less than the actual slant and so underestimation occurred. The amount of underestimation was in close agreement with the predictions of the model. No significant differences were found between the results for the real surface and those for the picture surface and so the picture stimuli were adopted in the experiments that followed.

Experiment 2 was designed to gain evidence for the primary mechanism of the underestimation model, namely that the perceived straight-ahead direction deviated from the true straight-ahead under the reduction conditions used in slant perception experiments. A zero-slanted surface was presented within a series of backward slanted surface presentations and the mean judged slant for this test surface was found to be significantly greater than zero and in a forward direction. This suggests that the perceived straight-ahead direction deviated from the true direction, presumably as a carry-over effect from the backward slanted surfaces that preceeded the test surface. For a forward slanted series, the zero-slant test surface was judged to be in a backward direction as predicted, but the result was not significant. When combined with other studies showing deviation of the perceived straight-ahead (MacDougall, 1903; Sharp, 1934; Kleinhans, 1970; Perrone, 1977), the results of experiment 2 indicate that deviation of the perceived straight-ahead could occur in the slant perception situation. The data gathered in experiment 2 for slant angles greater than zero degrees exhibited slant underestimation and once again this data was in close agreement with the predictions of the model.

Experiment 3 tested the generality of the model by using test surfaces made up of small texture units and a different aperture arrangement to that used in experiments 1 and 2. Even under these new conditions the amount of slant underestimation evident in the data was close to the predictions of the model.

Experiment 4, the main and final experiment, was a test of the model which examined the effect of specific changes to the parameters of the model upon slant estimates. Three different shaped rectanglar apertures were used which changed the amount and extent of the surface visible to the subject. A square, a wide rectangular and a narrow rectangular aperture were used. Test surfaces of 50° backward and 20° backward were viewed through these apertures and the model predicted an inverted u shaped function when judged slant is plotted against the three different aperture conditions. The data exhibited this predicted inverted u shaped function and supported the model.

## 9.2 Future Testing of the Model

A complete program designed to rigorously test all aspects of the model would include the following:

a) testing isolated figures covering a wide range of sizes. It should be possible to produce slant overestimation by careful selection of surface dimensions.

b) examining the minimum conditions required before deviation of the straight-ahead occurs. Which aspect of the reduction conditions produces this error?

c) testing the model at extreme angles, some of which include an 'horizon'. The experiments carried out so far included only a 50° slant as the maximum. This was because the size of the surface increases greatly for larger slants and the computation for production of the surfaces becomes excessive.

d) using textured surfaces with varying degrees of irregularity in order to discover any possible 'perspective extracting' strategies used by observers.

e) including stereoscopic presentation of test surfaces to test for any binocular/monocular differences.

f) using moving displays with 'velocity gradients' in order to extend the model to the dynamic case.

## 9.3 Summary of the Derivation of the Model

When an observer is presented with an isolated surface slanted relative to his/her line of sight and asked to make a judgement as to the extent of the slant, then the judged slant will more likely than not be less than the actual slant. This slant underestimation has been demonstrated in many studies under a variety of conditions, but it has never been explained adequately. Slant perception itself has always represented a major area in the general field of visual perception, involving theories of depth and distance perception, shape perception, size perception, spatial orientation and picture perception. Slant perception studies began in the 1950s with Gibson's (1950a) work, but slant underestimation per se has never been fully investigated, perhaps because it was overshadowed by the theoretical debates on visual space perception that arose from slant perception experiments.

I began by analysing the two-dimensional information at the eye and a mechanism was developed by which the slant angle could be extracted from this information. Two important variables formed the basis of this system; one was the projected length of part of the surface to one side of the pivot axis and the other was the angle of convergence of the perspective lines a certain distance out from the fixation point. It was shown how correct registration of these two variables would ensure the accurate perception of the angle of slant. A model was then developed which proposed that in the case of isolated slanted surfaces, instead of the two variables mentioned above, the total projected length of the surface and the angle of convergence of the outlines of the surface are used to extract the slant angle from the two-dimensional information. Given that this occurs we can calculate the slant angle defined by the information, on a purely geometrical basis, and an equation was developed for doing this. The predicted values for the slant estimates calculated from this equation are less than the true slant angle and thus the model predicts underestimation. The predicted values obtained from this equation were in close agreement with empirical data.

In order to explain why the observer uses the total projected length of the surface instead of the correct length to one side of the axis of rotation, it was suggested that given the reduction conditions that exist in slant perception experiments, the observer considers the perceived straight-ahead direction to lie in the direction of the nearest part of the surface. This may be the nearest edge of a slanted figure or it may be the direction of one edge of the viewing aperture, depending on the type of presentation. It was pointed out how this deviation of the perceived straight-ahead was a misapplication of a principle that exists in our normal visual environment, in which the straight-ahead direction in relation to a surface, coincides with the shortest distance to the surface.

Deviation of the perceived straight-ahead could account for the observer using the total projected length of the surface in the evaluation of the angle of slant. But even without an explanation as to why particular variables are used, the excellent correspondence between the predictions of the model and the empirical data suggest that the proposed mechanism for extracting the slant angle, may closely parallel that used by the human observer. The model can explain underestimation and it can account for specific results in given situations and so it has fulfilled the aims originally outlined at the start of this thesis.

## 9.4 Applying the Model to Past Experiments

The model also provides insight into some of the problems and unexplained results from the studies viewed in the first chapter, for example:

### 9.4.1 Shape-Slant Invariance Hypothesis

Most experiments testing the shape-slant invariance hypothesis have failed to find any conclusive evidence for it (e.g. Beck and Gibson, 1955;

Epstein, Botrager and Park, 1962; Clark, Smith and Rabe, 1955, 1956a, b; Smith, 1964).

The model provides an exact measure of the amount of slant that should be perceived in a particular situation and this value does not correspond to the slant value used to test the invariance hypothesis. Greater correspondence between shape and slant may result if perceived slant is based on the predictions of the model.

### 9.4.2 Braunstein's Results

It was pointed out in the introduction how Brainstein (1968) obtained very poor slant estimates for a textured surface, and it was noted that Brainstein had effectively modified the vertical extent of his surfaces depending on the slant angle used. In view of the principles of the model, we can now see that such a procedure would increase the value of L in equation [A] of the model and would result in a decrease in the slant estimate. This could explain Brainstein's (1968) result of only 11.6° mean judged slant for a 60° actual slant.

### 9.4.3 Freeman's Theory

Freeman (1965, 1966a, b) correctly pointed out that perspective information was important to slant perception and how this variable is dependent upon the field of view of the test surface. However Freeman (1965, 1966a, b) argued that judged slant should increase with the increased viewing area of a slanted surface. The research presented in this thesis has shown that this is not necessarily the case. Freeman recognised that perspective is greater for lines further out from the central axis of the surface, than those close to the central axis, and he tried to argue that the outlines of a figure will always produce greater slant judgements than texture information within the outlines.

However this is not a theory of underestimation and without the mechanism outlined in the model, veridical slant judgements will still result from correct registration of texture information within the outlines of the figure. Only when the perceptual errors described in the model are in effect, does the field of view of the surface become a factor in the slant judgements and Freeman never took this into account.

## 9.5 Application of the Principles of the Model to Other Areas

### 9.5.1 Picture Perception

Hagen and Glick (1977) in a study of pictorial perspective, tested the perception of size, linear perspective and texture perspective. They found that errors in size judgements occurred most frequently, indicating that the geometrically correct rate of perspective convergence was too rapid to be seen by the subjects as perceptually acceptable. Hagen and Glick (1977) indicated that for the case of ordinary pictures, it has been argued that frequently the convergence of traditional perspective is too extreme to look natural to the picture perceiver. Artists since the Renaissance have consistently modified perspective to correct this too sudden convergence. Zajac (1961) also wrote about this effect and commented on a difference that exists between perspective lines converging towards a point above them, and lines converging to a point below. Zajac (1961) commented on how the appearance of some paintings, which were painted with a strict adherence to geometrical laws of central perspective, looks unnatural because the convergence of parallel lines (especially those of the ceiling) seems to be much too pronounced. He quotes as examples the 'Interior of the Church' by Mansu Desiderio and 'Antwerp Cathedral' by Peter Neefs sen.

This effect has not been adequately explained and in fact it is only recently that analysis of pictorial space has been attempted (e.g. Farber and Rosinski, 1978; Goldstein, 1979). The most common approach is to examine the effect of changing the viewing position or station point. An application of the principles outlined in the underestimation model to picture perception problems, would be a fruitful area for future research, because a picture is effectively a limited portion of an optical array remote from the original reference axis of the depicted scene and shares many aspects of visual slant perception experiments. The effect noted by Hagen et al shows that illusions occur regarding perspective lines within pictures. If the depicted slant of the background is perceived incorrectly as outlined in the underestimation model, then the size perspective of superimposed objects will appear incorrect in relation to the background. This seems to agree with observations made by Hagen and Glick (1977).

### 9.5.2 Slope Misperception in the Natural Environment

Ross (1974), in a discussion of mountain road illusions, wrote that even when no illusory features are present, there is a strong tendency to overestimate the steepness of a frontal slope. The overestimation, according to Ross, increases at night and when viewed from a distance - conditions in which three-dimensional cues are much reduced. Ross (1974) also points out that on some occasions though, climbers and skiers frequently overestimate slopes in broad daylight, when in contact with the slope. 'Overestimation' in these cases refers to an overestimation of the steepness of a frontal slope. Using the convention to describe slant angles outlined in Chapter II this represents an underestimation of slant. The conditions in which the illusion occurs are conducive to the deviation of the perceived straight-ahead as outlined in the model. The steepness overestimation illusion may well be explainable by the underestimation model presented in Chapter III.

### 9.5.3 Pilot Error during Landings

Roscoe (1979) indicates some of the problems encountered by pilots using various types of imaging flight displays and visual systems for contact flight simulators. Roscoe reports that when pilots make approaches and landings with any type of imaging flight displays projected at unity magnification, they tend to come in fast and long, round out high and touch down hard. Roscoe (1979) suggests that these landing characteristics occur because the imaged runway appears smaller, further away and higher in the visual field than it does when viewed directly from the same approach path on a clear day. He outlined several other illusions that pilots are susceptible to in certain flying conditions and proposed an explanation based on the visual accommodation distance and changes to the perceived size of objects that can occur with changes to the accommodation of the eye.

The total situation in the pilot landing task is complex but there exist factors which parallel those involved in the perception of slanted surfaces, such as head-up viewing, limited field of view and the presence of perspective lines in the form of runway patterns. The judgement of ego-position by a pilot while landing is effectively a judgement of the angle of slant between his line of approach and the runway surface. Whether or not the principles outlined in the underestimation model can be applied to the problems encountered by pilots depends on which imaging system is being used and its specific properties. The application of the model to these problems is nevertheless, a potential area for future research.

### 9.5.4 Vestibular Cues

Experiments on slant perception have traditionally been carried out with the observer in an upright position with the fronto-parallel plane perpendicular to the ground plane. As has been pointed out previously, the process of making judgements of a pivoting surface while in an upright position, is an uncommon visual experience. In the case of backwards/forwards slant perception, it is more usual to orientate ourselves in relation to the two main horizontal planes (the ground and sometimes the ceiling). Left/right slant is not tied to the gravitational reference axis and usually occurs in relation to man-made features such as walls and fences.

Slant underestimation has been shown to occur in the case of left/right slant about a vertical axis as well as for backward/forward slant about a horizontal axis. So the fact that the observer views the surfaces while orientated parallel to the direction of gravity, should not be a factor in the underestimation. However, a task for future research would be to determine what role, if any, vestibular or kinaesthetic information plays in the perception of slant. The horizontal plane of the ground and the vertical direction may represent 'stable' directions for the perception of slant.

A pilot study was conducted in which a portable hand-held device, fitted with an eye-hole, an aperture and backlit transparencies of slanted surfaces, was set by subjects to a position that resulted in the apparent surface lying parallel to the ground plane. The complete unit was tilted along with the observers head and the ground was not visible. The individual differences were quite high, but these may have been due to an insufficient number of practice trials. However subjects did report that there was a position where the stimulus surface 'looked like the ground'. The author also observed this effect. At a particular position of the unit the apparent surface appeared to be as far away as the ground plane, (even though it was in fact only 25cm away and the ground was approximately 150cm away) as well as being parallel to it. This particular orientation of the unit 'felt right', although it was not always the correct position, given the depicted slant of the surface in the tilting unit. The errors tended to be such that the apparent surface was set at a position slanted downwards in relation to the ground plane. The slant was underestimated.

A similar effect was noted for the vertical direction and for a 'ceiling' condition. Just what the role of kinaesthetic information is in these situations is uncertain, but results from this pilot study are encouraging enough to consider further research with this technique. It may provide a more 'natural' means of testing slant perception since it involves changes to the orientation of the head in relation to a fixed reference plane, rather than the fixed head position and moving surface technique previously adopted.

### 9.5.5 Sensory Spatial After-Effects

Spatial after-effects in the third dimension of visual space were first demonstrated by Kohler and Emery (1947). They reported that a frontal-parallel cardboard strip was judged to be slanted in one direction following fixation of a similar strip slanted in the opposite direction. The similarity between this effect and the effect demonstrated in experiment 2 has already been noted. Kohler (1964) explained the spatial after-effect in terms of a cortical satiation hypothesis. However both this and opposing theories (e.g. Bergman and Gibson, 1959) have been shown to be inadequate as complete explanations of after-effect phenomenon (Wenderoth, 1970).

An explanation of slant after-effects based on deviation of the perceived straight-ahead has never been suggested, yet several reported observations indicate that this mechanism could form some part of the basis for the effect. Bergman and Gibson (1959) reported that every one of their observers spontaneously reported a straightening up of the apparent slant of the surface during his/her inspection period. Observers usually expressed surprise after about a minute, stating that the surface had become less slanted. This is in accord with the mechanisms outlined in Chapter III and it parallels the underestimation that occurs in standard slant perception experiments. The longer the period that the observer cannot see the reference axis of the room, the more complete the deviation of apparent straight-ahead. This could occur slowly over time in the same manner as so called 'adaptation' effects. We can presume that at the last stage of the inspection period, the observer's perceived straight-ahead direction is deflected towards the nearest part of the inspection surface. When presented with the test surface, this deflection persists and hence the surface will appear slanted in the opposite direction to the inspection surface producing the standard spatial after-effect. Consider as well, the effect described by Farne (1970). Farne noted with interest that the window or aperture he used to occlude the edges of his pre-test (inspection) surface, on some occasions appeared to have trapezoidal contours. 'The right side (corresponding to the farthest part of the surface) seems larger than the left one'. This is perfectly in accord with the mechanisms outlined in Chapter IV regarding the position of the apparent fronto-parallel plane and supports the idea that deviation of the apparent straight-ahead also occurs in spatial after-effect experiments.

Slant after-effects in the third dimension have been considered as a totally separate area from visual slant perception experiments. Gibson for instance, worked in both areas, yet he never connected the negative slant judgements obtained for his 10° surface slant (Gibson, 1950a) with the negative slant after-effect. I believe that sufficient evidence exists to suggest that the same mechanism that results in visual slant underestimation can be used to explain the slant after-effect.

## 9.6 Epilogue

Gibson (1950a, b) believed that there was a one-to-one correspondence between perception and geometric parameters of the optical stimulus and he chose to examine the variable of texture gradients. The resulting poor correspondence between the physical slant and the perceived slant, strengthened the arguments put forward by opponents to his theories. The material presented in this thesis indicates that it was an aspect of the experimental situation adopted by Gibson (1950a) that led to the poor psychophysical correspondence. The correspondence between the geometrical properties of the optical stimulus and the perceived slant has been shown in the studies reported, to be excellent. However, the perception of slant appears to be based on the wrong features of the stimulus because the experimental situation presents the information in a form not normally encountered in the environment. Had Gibson realised this, his psychophysical theory may have been more readily accepted.

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1.

```
5 C PROGRAM FOR RECTANGULAR
6 C FIGURES.
7 C CALCULATES BETA,
8 C PREDICTED SLANT.
10 INPUT"DISTANCE";D
20 INPUT"HALF LENGTH";L
30 INPUT"WIDTH";W
40 CH=.0174533
50 INPUT"THETA=..?(100 FOR STOP)
";T
60 A=T*CH
70 IF T=100 GOTO 130
80 K=4*L*D*D*COS(A)
90 M=W*TAN(A)*(D+L*SIN(A))*(D-L*
SIN(A))
100 B=ATN(M/K)
110 LPRINT T;TAB(10)B/CH
120 GOTO  50
130 LPRINT "D= ";D;TAB(5)"L= ";L
;TAB(5)"W= ";W
135 LPRINT"MODEL 2 FOR ISOLATED
FIGURES"
140 STOP
150 END
```

2.

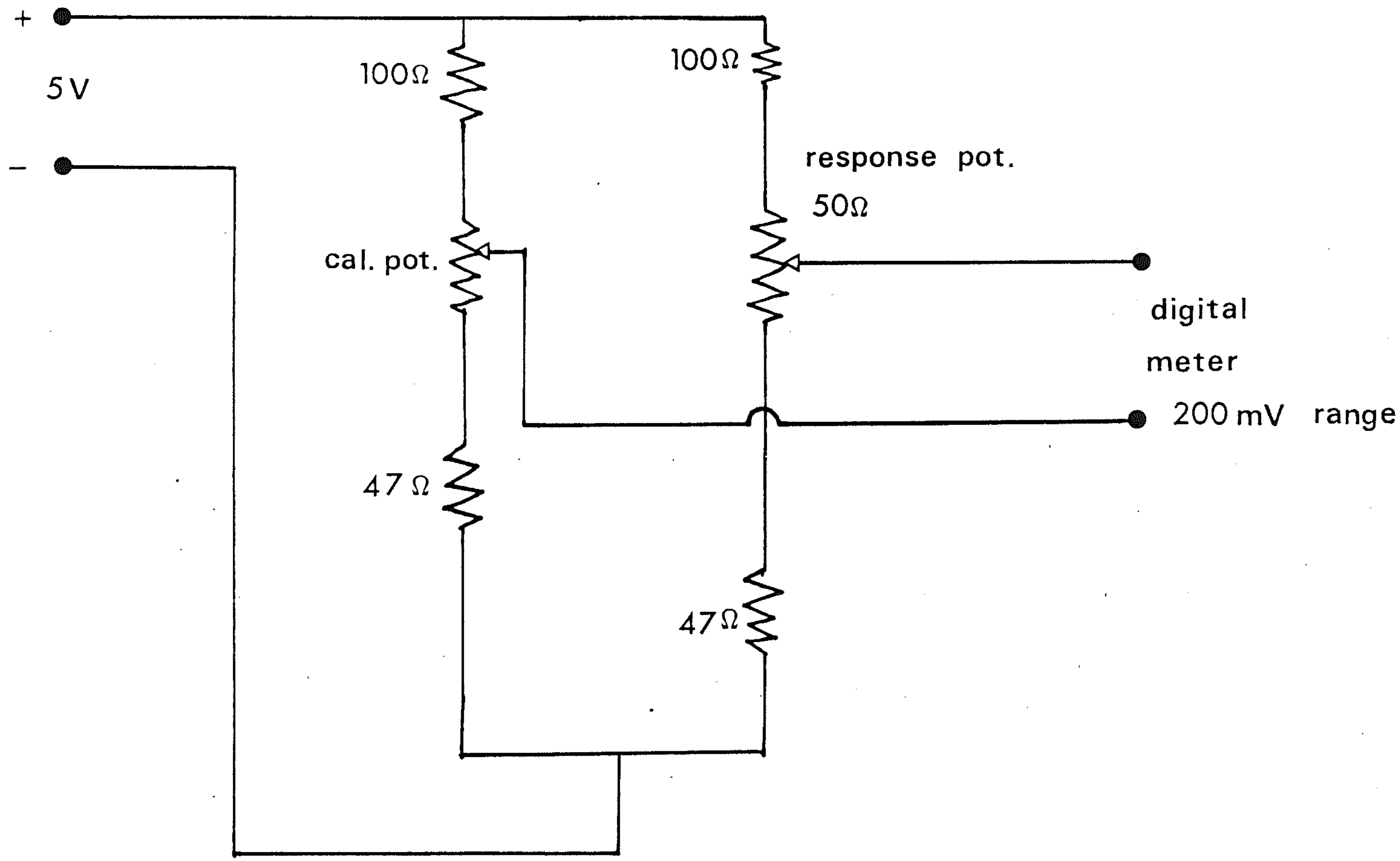
```
1 C PROGRAM FOR
2 C CIRCULAR  APERTURES
10 INPUT"DISTANCE";D
20 INPUT "FIELD OF VIEW";F
30 CH=.0174533
40 V=(F/2)*CH
50 B=4*D*TAN(V)
80 INPUT"WIDTH OF SMALLEST UNIT"
;W
90 X=D*TAN(V)
110 W1=W
120 IF W1>X GOTO 150
130 W1=W1+W
140 GOTO 120
150 W2=W1-W
160 IF W2=0 GOTO 200
170 INPUT"THETA..100 FOR STOP";A

175 IF A=100 GOTO 250
180 A1=A*CH
190 T=(TAN(A1)*2*W2)/B
192 B1=ATN(T)
195 GOTO 220
200 PRINT"NO LINES VISIBLE"
210 GOTO 250
220 LPRINT A;TAB(5)"BETA";B1/CH;
TAB(10)"W"2*W2;D;F
240 GOTO 170
250 STOP
260 END
```

3.

```
5 C PROGRAM FOR
6 C RECTANGULAR APERTURES
10 PRINT"RECTANGULAR APERTURES"
20 INPUT"DISTANCE";D
30 INPUT"VERTICAL FIELD OF VIEW"
;F
40 CH=.0174533
50 V=(F/2)*CH
60 B=4*D*TAN(V)
70 INPUT"HORIZONTAL FIELD OF VIE
W";H
80 INPUT "WIDTH OF SMALLEST UNIT
";W
90 I=(H/2)*CH
100 INPUT"THETA..100=STOP";A
105 IF A=100 GOTO 260
110 A1=A*CH
120 X=(D*TAN(I)*COS(A1))/COS(A1+
V)
130 W1=W
140 IF W1>X GOTO 170
150 W1=W1+W
160 GOTO 140
170 W2=W1-W
180 IF W2=0 GOTO 220
190 T=(TAN(A1)*2*W2)/B
200 B1=ATN(T)
210 GOTO 240
220 PRINT"NO LINES VISIBLE"
230 GOTO 260
240 LPRINT A;TAB(5)"BETA";B1/CH;
TAB(10)"W";2*W2;D;F;H
250 GOTO 100
260 STOP
270 END
```

```
$ SET AUTOBIND
$ BIND=FROM PLOTA/=
FILE  1(TITLE='XVALUES',KIND=DISK,FILETYPE=7)
FILE  2(TITLE='FROW',KIND=DISK,FILETYPE=7)
FILE  3(TITLE='EYEPOS',KIND=DISK,FILETYPE=7)
FILE  11(TITLE='NXLIMIT',KIND=DISK,FILETYPE=7)
FILE  12(TITLE='BOXSIZE',KIND=DISK,FILETYPE=7)
FILE  4(TITLE='ANGLES',KIND=DISK,FILETYPE=7)
FILE  14(TITLE='LABELS',KIND=DISK,FILETYPE=7)
DIMENSION XD(400),YD(400),ATITLE(1)
 DIMENSION X1(500),Y1(500),Z1(500),X(500),Y(500),W(50)
 READ(11,10,END=601)NX,LIMIT
 10FORMAT(2I3)
 PRINT */,NX,LIMIT
 NRUN=0
 5READ(12,12,END=601)WIDTH,HEIGHT
 12FORMAT(2F5.0)
 DO 35 I=1,NX
  READ(1,/)W(I)
  35 CONTINUE
  37READ(2,36,END=601)FROW,YINC,Z,GRAD,TOTAL
  36FORMAT(5F5.0)
  PRINT */,FROW,YINC,Z,GRAD,TOTAL
  J=1
  ROWS=0.0
  50 DO 60 I=1,NX
  X1(J)=W(I)
  Y1(J)=FROW
  Z1(J)=Z
  60 J=J+1
  FROW=FROW+YINC
  Z=Z+GRAD
  ROWS=ROWS+1
  IF(ROWS.GE.TOTAL)GO TO 70
  GO TO 50
  70READ(3,71,END=601)C1,C2,C3,D
  71FORMAT(4F5.0)
 READ(4,72,END=601)R1,R2,R3
  72FORMAT(3F7.0)
  N=J-1
  WRITE(6,100)N
  100 FORMAT(' TOTAL N ',I6)
  PRINT */,C1,C2,C3,D
  PRINT */,R1,R2,R3
C* *****PROJECT*********
  I=1
  Q1=C1+D*COS(R1)
  Q2=C2+D*COS(R2)
  Q3=C3+D*COS(R3)
  DO 250 J=1,N
  CR1=(X1(J)-C1)*COS(R1)
  CR2=(Y1(J)-C2)*COS(R2)
  CR3=(Z1(J)-C3)*COS(R3)
  CRATIO=D/(CR1+CR2+CR3)
  P1=C1+CRATIO*(X1(J)-C1)
  P2=C2+CRATIO*(Y1(J)-C2)
  P3=C3+CRATIO*(Z1(J)-C3)
  AI1=(P1-Q1)*COS(R2)
  AI2=(P2-Q2)*COS(R1)
  AI3=AI1+AI2
  XSUB=AI3/SIN(R3)
  YSUB=(P3-Q3)/SIN(R3)
  X(I)=XSUB
  Y(I)=YSUB
  250 I=I+1
  CALL AINIT(1000)
  CALL ASPEED(0)
  CALL AORIG(500,500)
  520 J=1
  DO 525 I=1,N
  IF(ABS(X(I)).LE.WIDTH/2.0)GO TO 521
  GO TO 525
  521 IF(ABS(Y(I)).LE.HEIGHT/2.0) GO TO 522
  GO TO 525
  522 XD(J)=X(I)
  YD(J)=Y(I)
  J=J+1
  525 CONTINUE
  DATA CHAR/'.'/
  N=J-1
  CALL ALINEC(XD,YD,N,0,0,.0254,.0254,CHAR,0,0,1,2)
  CONV=(-WIDTH/2.0)*3937.0
  540 IJX=INT(CONV+.5)
 IF(IJX.LT.-500) GO TO 550
  CONY=(-HEIGHT/2.0)*3937.0
  IJY=INT(CONY+.5)
 IF(IJY.LT.-500)GO TO 550
  COF=WIDTH*3937.0
  JINX=INT(COF+.5)
  CCF=HEIGHT*3937.0
  JINY=INT(CCF+.5)
 CALL ABOX(IJX,IJY,1,1,JINX,JINY,4)
 READ(14,541)ATITLE
  541FORMAT(A6)
 CALL ALAB(-450,-480,ATITLE,6,2,2)
  GO TO 599
  550 WRITE(6,553)
  GO TO 599
  551 WRITE(6,554)
  553 FORMAT(' FRAME TOO BIG ')
  554 FORMAT(' LABEL FILE RAN OUT ')
  599 CALL AEND
  NRUN=NRUN+1
  IF(NRUN.LT.LIMIT)GO TO 37
  WRITE(6,600)ATITLE
  600FORMAT(1X,' LABEL= ',A6)
  WRITE(6,604)
  604FORMAT(1X,' END OF RUN ')
  GO TO 605
  601 WRITE(6,602)
  602FORMAT(1X,' FILE RAN OUT ')
  605 STOP
  END
#
```


+
5V
−
100Ω
100Ω
response pot.
50Ω
cal. pot.
digital
meter
200 mV range
47Ω
47Ω

# APPENDIX D

Experiment 1

10°

| Source | Error Term | SS | df | MS | F |
|---|---|---|---|---|---|
| A | AS | 2.116 | 1 | 2.116 | 1.8581 |
| B | BS | 6.084 | 1 | 6.084 | 1.7516 |
| S | | 175.795 | 9 | 19.533 | |
| AB | ABS | 0.256 | 1 | 0.256 | 0.1082 |
| AS | | 10.249 | 9 | 1.139 | |
| BS | | 31.261 | 9 | 3.473 | |
| ABS | | 21.299 | 9 | 2.367 | |

30°

| Source | Error Term | SS | df | MS | F |
|---|---|---|---|---|---|
| A | AS | 1.089 | 1 | 1.089 | 0.288 |
| B | BS | 47.089 | 1 | 47.089 | 3.214 |
| S | | 869.330 | 9 | 96.592 | |
| AB | ABS | 3.136 | 1 | 3.136 | 0.637 |
| AS | | 34.031 | 9 | 3.781 | |
| BS | | 131.861 | 9 | 14.651 | |
| ABS | | 44.304 | 9 | 4.923 | |

50°

| Source | Error Term | SS | df | MS | F |
|---|---|---|---|---|---|
| A | AS | 0.600 | 1 | 0.600 | 0.036 |
| B | BS | 234.740 | 1 | 234.740 | 3.592 |
| S | | 1212.707 | 9 | 134.745 | |
| AB | ABS | 15.006 | 1 | 15.006 | 2.242 |
| AS | | 150.967 | 9 | 16.774 | |
| BS | | 588.097 | 9 | 65.344 | |
| ABS | | 60.231 | 9 | 6.692 | |

Experiment 2

10°

| Source | Error Term | SS | df | MS | F |
|---|---|---|---|---|---|
| A | AS | 0.200 | 1 | 0.200 | 0.041 |
| S | | 23.888 | 9 | 2.654 | |
| AS | | 44.320 | 9 | 4.924 | |

20°

| Source | Error Term | SS | df | MS | F |
|---|---|---|---|---|---|
| A | AS | 29.282 | 1 | 29.282 | 11.444 |
| S | | 92.962 | 9 | 10.329 | |
| AS | | 23.028 | 9 | 2.559 | |

30°

| Source | Error Term | SS | df | MS | F |
|---|---|---|---|---|---|
| A | AS | 77.225 | 1 | 77.225 | 5.058 |
| S | | 249.831 | 9 | 27.759 | |
| AS | | 137.411 | 9 | 15.268 | |

40°

| Source | Error Term | SS | df | MS | F |
|---|---|---|---|---|---|
| A | AS | 191.581 | 1 | 191.581 | 9.401 |
| S | | 530.823 | 9 | 58.980 | |
| AS | | 183.415 | 9 | 20.379 | |

50°

| Source | Error Term | SS | df | MS | F |
|---|---|---|---|---|---|
| A | AS | 161.312 | 1 | 161.312 | 4.562 |
| S | | 532.008 | 9 | 59.112 | |
| AS | | 318.208 | 9 | 35.356 | |

Experiment 3

10°

| Source | Error Term | SS | df | MS | F |
|---|---|---|---|---|---|
| A | AS | 1.891 | 1 | 1.891 | 0.046 |
| S | | 31.049 | 7 | 4.436 | |
| AS | | 286.084 | 7 | 40.869 | |

20°

| Source | Error Term | SS | df | MS | F |
|---|---|---|---|---|---|
| A | AS | 14.251 | 1 | 14.251 | 0.218 |
| S | | 131.149 | 7 | 18.736 | |
| AS | | 457.504 | 7 | 65.358 | |

30°

| Source | Error Term | SS | df | MS | F |
|---|---|---|---|---|---|
| A | AS | 74.391 | 1 | 74.391 | 2.615 |
| S | | 283.929 | 7 | 40.561 | |
| AS | | 199.104 | 7 | 28.443 | |

50°

| Source | Error Term | SS | df | MS | F |
|---|---|---|---|---|---|
| A | AS | 306.250 | 1 | 306.250 | 15.749 |
| S | | 446.050 | 7 | 63.721 | |
| AS | | 136.120 | 7 | 19.446 | |